1.5

One-to-One Fnc

- passes VLT & HLT
- every output is associated w/ exactly one input.

Official Def:
f(x) is one-to-one on a domain D if $f(a) \neq f(b)$ when $a \neq b$.

We care: the inv. of a one-to-one fnc is also a fnc.

Aug 27-1:04 PM

Inv. Fnc

what: reversing a one-to-one fnc.

$f^{-1}(x)$

how to find: switch x & y, solve y

Algebraic:
f(x) & g(x) are inverses iff
$(f \circ g)(x) = (g \circ f)(x) = x$

Graphical:
f(x) & g(x) are inv. iff they are reflection of eachother over y=x.

Aug 27-1:16 PM

1. $f(x) = -(x-2)^2 \quad x \leq 2$

$x = -(y-2)^2 \quad y \leq 2$

$-1 \qquad -1$

$\sqrt{-x} = \sqrt{(y-2)^2}$

$y - 2 = \pm\sqrt{-x}$

$+2 \quad +2$

$y = 2 \pm \sqrt{-x}, \; y \leq 2$

Aug 27-1:19 PM

2. $g(x) = \dfrac{x+4}{2x+1}$

$(2y+1)(x) = \left(\dfrac{y+4}{2y+1}\right)(2y+1)$

$2xy + x = y + 4$

$-y \quad -x \qquad -y \quad -x$

OR

$2xy + x = y + 4$

$-2xy \quad -4 \qquad -2xy \quad -4$

$2xy - y = -x + 4$

$x - 4 = -2xy + y$

$\dfrac{y(2x-1)}{2x-1} = \boxed{\dfrac{-x+4}{2x-1}}$ $\longleftrightarrow$ $\boxed{\dfrac{x-4}{-2x+1} = \dfrac{y(-2x+1)}{-2x+1}}$

equivalent

Aug 27-1:22 PM

3. $f(x) = 3^{x+2}$

$\ln x = \ln 3^{(y+2)}$

$\frac{\ln x}{\ln 3} = \frac{(y+2)\ln 3}{\ln 3}$

$y+2 = \frac{\ln x}{\ln 3}$

$-2 \quad -2$

$y = \frac{\ln x}{\ln 3} - 2$

OR $x = 3^{y+2}$

Change to log. form

$\log_3 x = y+2$

$-2 \quad -2$

$*y = \log_3 x - 2$

Aug 27-1:26 PM

RS log. form ⟷ exp. form

15. $\log_b y = x \longleftrightarrow b^x = y$

17. $\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$

18. $\log_b xy = \log_b x + \log_b y$

19. $\log_b \frac{x}{y} = \log_b x - \log_b y$

20. $\log_b x^r = r \log_b x$

21. $\log_b \sqrt[n]{x} = \frac{1}{n} \log_b x$

Aug 27-1:28 PM

4. $\frac{4e^{.05t}}{4} = \frac{17}{4}$

$\ln e^{.05t} = \ln \frac{17}{4}$

$.05t \ln e = \ln \frac{17}{4}$

$\frac{.05t}{.05} = \frac{\ln \frac{17}{4}}{.05}$

$t \approx 28.938$

Aug 27-1:32 PM

5. $(e^x + e^{-x} = 3)e^x$

$(e^x)^2 + e^0 = 3e^x$

$(e^x)^2 - 3e^x + 1 = 0$ Let $w = e^x$

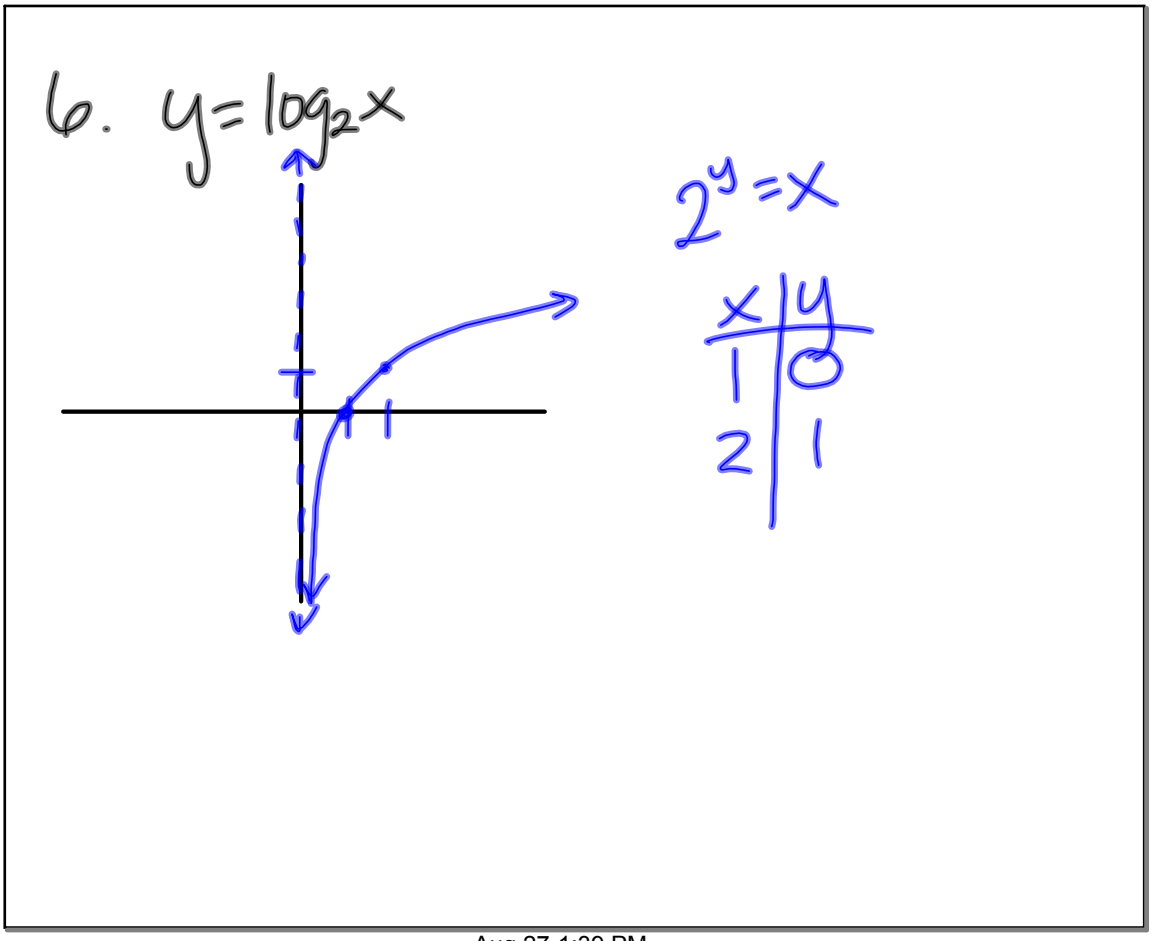
$w^2 - 3w + 1 = 0$

$w = \frac{3 \pm \sqrt{9-4(1)(1)}}{2} = \frac{3 \pm \sqrt{5}}{2}$

$\ln e^x = \ln \frac{3 \pm \sqrt{5}}{2}$

$x = \ln \left(\frac{3 \pm \sqrt{5}}{2} \right)$

Aug 27-1:35 PM



Aug 27-1:39 PM