1.5 Parametric Relations & Inverses

Obj: 1. Define fncs & relations parametrically.

2. Find inverses of fncs & relations.

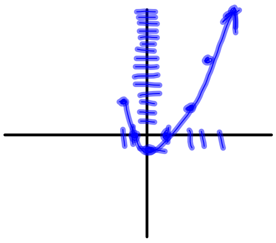
Parametric:

$$x = t+1$$
$$y = t^2+2t$$

t is any real #.

Find points (x,y) when $t=-3,-2,-1,0,1,2,3$

x	y	t
-2	3	-3
-1	0	-2
0	-1	-1
1	0	0
2	3	1
3	8	2
4	15	3



Sep 20-9:39 AM

Find an algebraic relationship between x and y: get rid of t "eliminating the parameter"

$$*\begin{cases} x = t+1 \\ y = t^2+2t \end{cases}$$

1. Solve x-eq. for t.

$$x-1 = t$$

2. Plug in t-value to the the y eq.

$$y = (x-1)^2 + 2(x-1)$$
$$y = (x-1)(x-1) + 2x - 2$$
$$y = x^2 \cancel{-2x} + 1 \cancel{+2x} - 2$$
$$*y = x^2 - 1$$

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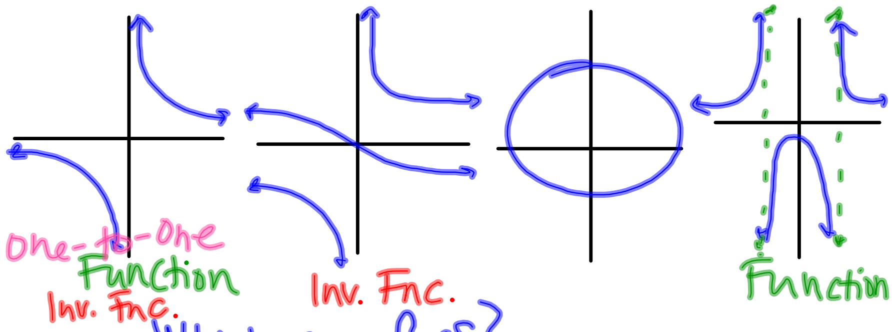
Functions / Relations / Inverses



Which are fncs?

Pass the VLT

Which have inverses that are fncs?

Pass The HLT.

If a graph pass The HLT & VLT (it is fnc & the inv. is a fnc) ∴ one-to-one

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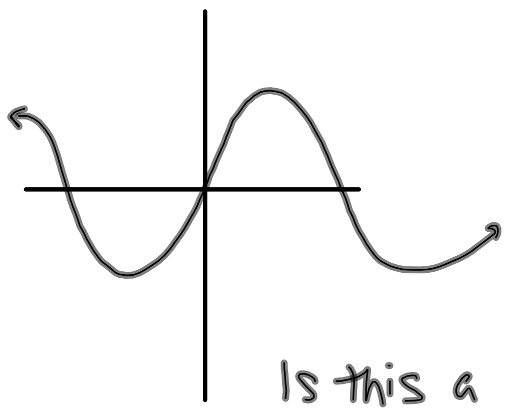

Is this a fnc? Yes

Is its inv a fnc: NO!

Not one-to-one

Sep 20-10:07 AM

Inverse: switch domain & range.

$f(x) \longrightarrow$ inverse: $f^{-1}(x)$

Find an eq. for $f^{-1}(x)$ if $f(x) = \frac{x}{x+1}$.

1. Switch x & y:

$(y+1)(x) = \left(\frac{y}{\cancel{y+1}}\right)\cancel{y+1}$

2. Solve for y:

$\cancel{xy} + x = y - xy$
$\cancel{-xy}$

$\frac{x}{1-x} = \frac{y(1-\cancel{x})}{\cancel{1-x}}$

$f^{-1}(x)$
$y = \frac{x}{1-x}$

D: $1-x \neq 0$
$x \neq 1$

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Find $f^{-1}(x)$ if $f(x) = \frac{x+3}{x-2}$ & find domain.

$(y-2)(x) = \left(\frac{y+3}{\cancel{y-2}}\right)\cancel{(y-2)}$

$xy - 2\cancel{x} = \cancel{y} + 3$
$-y \quad +2x \quad -y \quad +2x$

$xy - y = 2x+3$

$\frac{y(x-1)}{x-1} = \boxed{\frac{2x+3}{x-1}} \cdot \frac{-1}{-1} = \frac{-2x-3}{-x+1}$

$-2x-3 = y - xy$

$\boxed{\frac{-2x-3}{1-x}} = \frac{y(1-\cancel{x})}{\cancel{1-x}}$

$=$

D: $-x+1 \neq 0$
$x \neq 1$

Sep 20-10:12 AM

Find $f^{-1}(x)$ if $f(x) = \sqrt{x}$ $D: x \geq 0$

$$x^2 = \sqrt{y}^2 \quad y \geq 0$$

$$y = x^2 \quad y \geq 0$$

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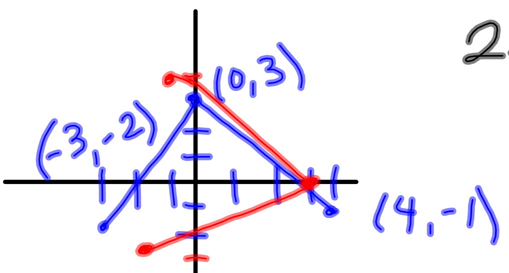
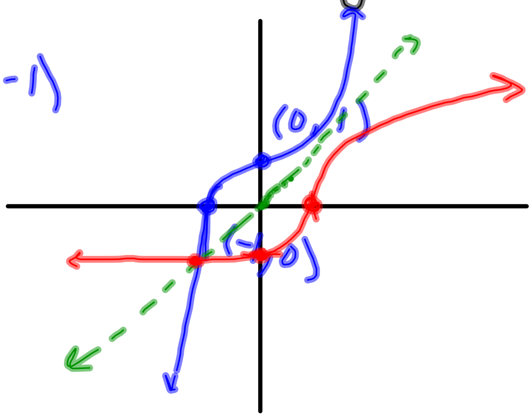
Graphically: 1. Swith x & y

2. Reflect over the line $y = x$



Inv: $(-2,-3)$
$(3,0)$
$(-1,4)$



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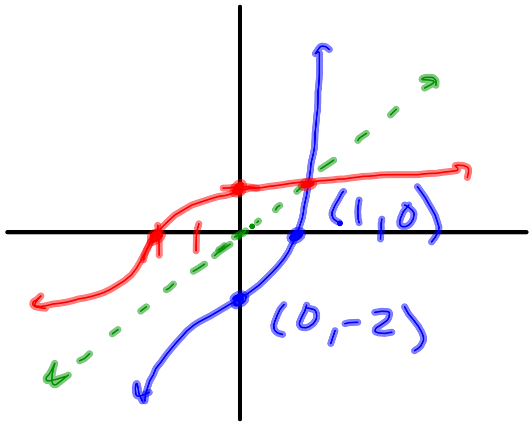
Find $f^{-1}(x)$ graphically:



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Inverse Composition Rule:

$f(x)$ and $g(x)$ are inverses iff

$f(g(x)) = x$

$g(f(x)) = x$

Verify
Show that $f(x) = x^3 + 1$ $g(x) = \sqrt[3]{x-1}$ are inverses.

$f \circ g = \left(\sqrt[3]{x-1}\right)^3 + 1$
$= x \cancel{-1} \cancel{+1} = x$

$g \circ f = \sqrt[3]{x^3 \cancel{+1} \cancel{-1}} = \sqrt[3]{x^3} = x$

✓

Sep 20-10:26 AM

Verify using composition of fncs that

$f(x) = \frac{x+3}{4}$ $g(x) = 4x - 3$ are inverses.

$f(g(x)) = \frac{(4x \cancel{-3}) \cancel{+3}}{4} = \frac{\cancel{4}x}{\cancel{4}} = x$

$g(f(x)) = \cancel{4}\left(\frac{x+3}{\cancel{4}}\right) - 3 = x \cancel{+3} \cancel{-3} = x$

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