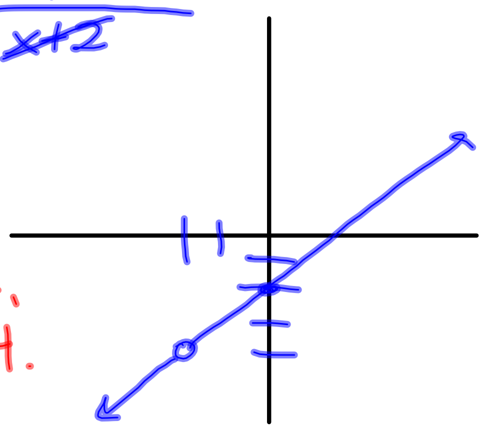
2.1 Limits

$$f(x) = \frac{x^2-4}{x+2} = \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}}$$

$x \neq -2$ Hole



As x gets closer to -2,
y gets closer to -4.

$$\lim_{x \to -2} f(x) = -4$$

"The limit of f(x) as x approaches -2 is -4"

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Analytically:

$$\lim_{x \to -3} \frac{x+3}{x^2+x-6}$$

1. Substitution: $\frac{-3+3}{9-3-6} = \frac{0}{0}$ ☹

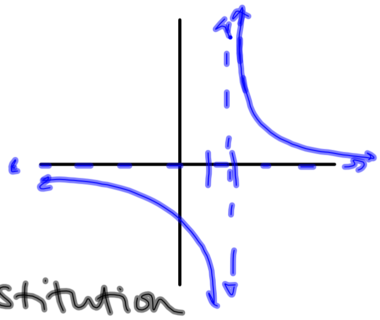
2. Simplify (then substitute)

$$\lim_{x \to -3} \frac{\cancel{x+3}}{\cancel{(x+3)}(x-2)} = \lim_{x \to -3} \frac{1}{x-2} = \frac{1}{-3-2} = -\frac{1}{5}$$

Graphically:

$$f(x) = \frac{1}{x-2}$$

$$\lim_{x \to \infty} f(x) = 0$$



Numerically: Substitution

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1. $\lim\limits_{x\to 2} x^2+3x-5 = 4+6-5 = 5$

2. $\lim\limits_{x\to \pi} (\sin x \cos x) = \sin\pi\cos\pi = 0\cdot -1 = 0$

3. $\lim\limits_{x\to 4} \frac{x-4}{x^2-16} = \lim\limits_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(x+4)} = \frac{1}{8}$

4. $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ MEMORIZE

5. $\lim\limits_{x\to 0} \frac{\sin x + x}{x} = \lim\limits_{x\to 0}\left(\frac{\sin x}{x} + \frac{\cancel{x}}{\cancel{x}}\right)$

$1+1=2$

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Properties

1. $\lim\limits_{x\to a}\left(f(x)\pm g(x)\right) = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$

2. $\lim\limits_{x\to a}\left(f(x)\cdot g(x)\right) = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x)$

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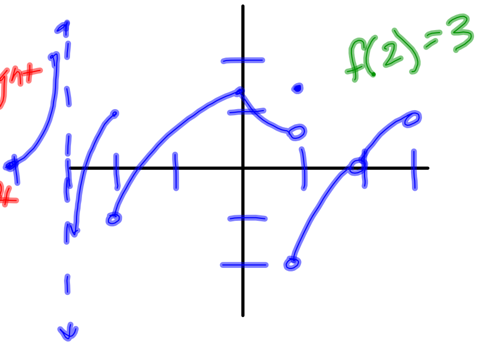
6. $\lim_{x\to 2^+} f(x) = -4$ from the right

7. $\lim_{x\to 2^-} f(x) = 1$ from the left

8. $\lim_{x\to 4^+} f(x) = 0$

9. $\lim_{x\to 4^-} f(x) = 0$



* The limit exists if LH & RH limit are =.

$\lim_{x\to 4} f(x) = 0$ $\lim_{x\to 2} f(x) = DNE$

Definition:

If $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$, then $\lim_{x\to a} f(x) = L$.

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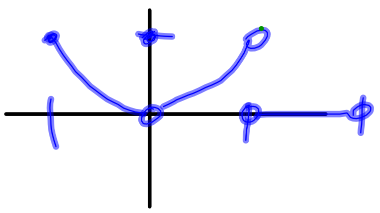

a. $\lim_{x\to -1^+} f(x) = 1$ T

b. $\lim_{x\to 0^-} f(x) = 0$ T

c. $\lim_{x\to 0^-} f(x) = 1$ F

d. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$ T

e. $\lim_{x\to 0} f(x)$ exists T

f. $\lim_{x\to 0} f(x) = 0$ T

g. $\lim_{x\to 0} f(x) = 1$ F

h. $\lim_{x\to 1} f(x) = 1$ F

* LH lim ≠ RH lim

i. $\lim_{x\to 1} f(x) = 0$ F

j. $\lim_{x\to 2^-} f(x) = 2$ F

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