2.4 Rates of Change

$y = 1.86t^2$

a. $[0,2]$

$$\frac{\Delta y}{\Delta t} = \frac{f(2) - f(0)}{2-0} = \frac{1.86(2)^2 - 0}{2-0} = 3.72 \text{ m/s}$$

b. $[1,4] = \frac{f(4)-f(1)}{4-1} = 9.3 \text{ m/s}$

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c. ~~Instantaneous~~ *Rate of Change Speed:

* try to find the average speed on a really small interval.

$[3, 3.1]$ (3+.1) $\quad \frac{f(3.1) - f(3)}{3.1-3} = 11.346 \text{ m/s}$

$[3, 3.01]$ (3+.01) $\quad \frac{f(3.01) - f(3)}{3.01-3} = 11.1786 \text{ m/s}$

$[3, 3+h] \quad \lim_{h \to 0} \frac{f(3+h) - f(3)}{(3+h)-3} = \lim_{h \to 0} \frac{1.86(3+h)^2 - 1.86(3)^2}{h}$

$$\lim_{h \to 0} \frac{1.86(9+6h+h^2) - 16.74}{h}$$

$$\lim_{h \to 0} \frac{\cancel{16.74} + 11.16h + 1.86h^2 \cancel{-16.74}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(11.16 + 1.86h)}{\cancel{h}}$$

$= 11.16 \text{ m/s}$

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Average Rate of Change:

Slope of the secant line: $[a, b]$

$$\frac{f(b) - f(a)}{b - a}$$

Instantaneous Rate of Change:

Slope of the tangent line

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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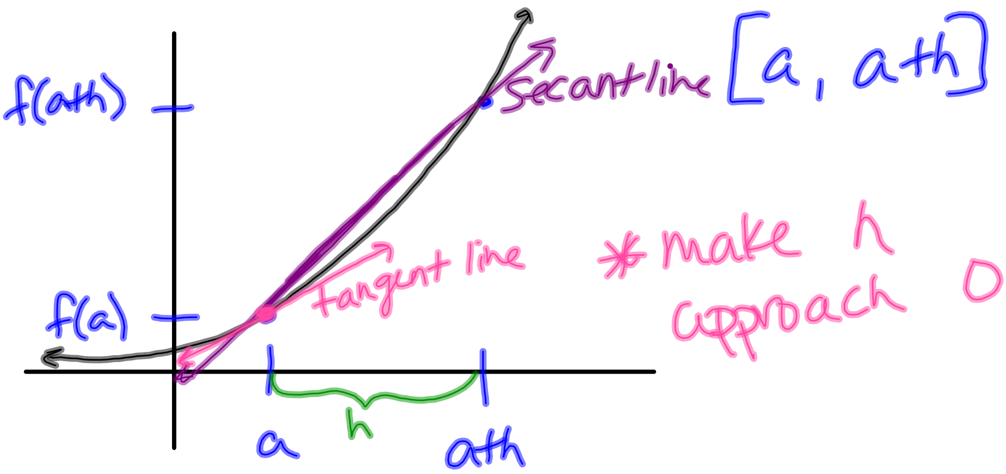

Difference Quotient Thm:

Slope of the tangent line @ $x = a$ is:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

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1. $f(x)=\frac{1}{4}x^2 \quad [1,3] \quad a=1$

a. $\frac{f(3)-f(1)}{3-1}=\frac{\frac{9}{4}-\frac{1}{4}}{2}=\frac{2}{2}=1$

b. $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{\frac{1}{4}(1+h)^2-\frac{1}{4}}{h}$

$\lim_{h\to 0}\frac{\frac{1}{4}(1+2h+h^2)-\frac{1}{4}}{h}=\lim_{h\to 0}\frac{\cancel{\frac{1}{4}}+\frac{1}{2}h+\frac{1}{4}h^2\cancel{-\frac{1}{4}}}{h}$

$\lim_{h\to 0}\frac{\frac{1}{2}h+\frac{1}{4}h^2}{h}=\lim_{h\to 0}\frac{\cancel{h}(\frac{1}{2}+\frac{1}{4}h)}{\cancel{h}}=\frac{1}{2}$

c. $m=\frac{1}{2}$ @ $x=1$

x	y
1	$\frac{1}{4}$

$y=\frac{1}{2}(x-1)+\frac{1}{4}$

d. normal line: $\perp$ to the tangent line.

$y=-2(x-1)+\frac{1}{4}$

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2. $f(x)=\frac{1}{x} \quad [-2,-1] \quad a=2$

a. $\frac{f(-1)-f(-2)}{-1+2}=\frac{-1+\frac{1}{2}}{1}=-\frac{1}{2}$

b. $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{\frac{2}{2}\left(\frac{1}{2+h}\right)-\left(\frac{1}{2}\right)\frac{(2+h)}{(2+h)}}{h}$

$\lim_{h\to 0}\frac{\frac{2-2-h}{2(2+h)}}{h}=\lim_{h\to 0}\frac{-\cancel{h}}{4+2h}\cdot\frac{1}{\cancel{h}}$

$\lim_{h\to 0}\frac{-1}{4+2h}=-\frac{1}{4}$

c. $m: -\frac{1}{4}$ @ $x=2$

x	y
2	$\frac{1}{2}$

$y=-\frac{1}{4}(x-2)+\frac{1}{2}$

d. $y=4(x-2)+\frac{1}{2}$

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3. $f(x) = x^2 - x$ $[0, 4]$ $a = 3$

a. $\dfrac{f(4) - f(0)}{4-0} = \dfrac{12-0}{4} = 3$

b. $\lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \dfrac{(3+h)^2 - (3+h) - (3^2 - 3)}{h}$

$\lim_{h \to 0} \dfrac{\cancel{9} + 6h + h^2 - \cancel{3} - h - \cancel{6}}{h} = \lim_{h \to 0} \dfrac{5h + h^2}{h}$

$\lim_{h \to 0} \dfrac{\cancel{h}(5+h)}{\cancel{h}} = 5$

c. $m: 5$ @ $x = 3$

x	y
3	6

$y = 5(x-3) + 6$

d. $y = -\frac{1}{5}(x-3) + 6$

$y = m(x - x_1) + y_1$

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4. $f(x) = -2x^2 + 1$ $x = a$

$\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$\lim_{h \to 0} \dfrac{-2(a+h)^2 + 1 - (-2a^2 + 1)}{h}$

$\lim_{h \to 0} \dfrac{-2(a^2 + 2ah + h^2) + 1 + 2a^2 - 1}{h}$

$\lim_{h \to 0} \dfrac{\cancel{-2a^2} - 4ah - 2h^2 \cancel{+1} \cancel{+2a^2} \cancel{-1}}{h}$

$\lim_{h \to 0} \dfrac{-4ah - 2h^2}{h} = \lim_{h \to 0} \dfrac{\cancel{h}(-4a - 2h)}{\cancel{h}} = -4a$

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5. $f(x) = 9 - 3x^2 \quad x = a$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h \to 0} \frac{9 - 3(a+h)^2 - (9 - 3a^2)}{h}$$

$$\lim_{h \to 0} \frac{9 - 3(a^2 + 2ah + h^2) - 9 + 3a^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{9} - \cancel{3a^2} - 6ah - 3h^2 \cancel{- 9} + \cancel{3a^2}}{h}$$

$$\lim_{h \to 0} \frac{-6ah - 3h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(-6a - 3h)}{\cancel{h}} = -6a$$

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