2.4 Real Zeros of Polynomial Fncs

Obj: 1. Use Long Division & Division Algorithm

2. Remainder & Factor Thms.

3. Synthetic Division

Review:

quotient

$$\begin{array}{r} 112 + \frac{3}{32} \\ 32\,\overline{)\,3587} \\ -32 \\ \hline 38 \\ -32 \\ \hline 67 \\ -64 \\ \hline 3 \end{array}$$

divisor dividend

remainder

Oct 5-2:28 PM

$$\begin{array}{r} x^2 + x + 2 + \frac{3}{3x+2} \\ 3x+2\,\overline{)\,3x^3+5x^2+8x+7} \\ -(3x^3+2x^2) \\ \hline 3x^2+8x \\ -(3x^2+2x) \\ \hline 6x+7 \\ -(6x+4) \\ \hline 3 \end{array}$$

Summary Statement / Polynomial Form:

(Divisor)(Quotient) + Remainder = Dividend

$$32\,\overline{)\,3587} \quad 112 + \frac{3}{32}$$

$$32 \times 112 + 3 = 3587$$

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Division Algorithm:

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend divisor quotient remainder

Fraction Form:

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

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Use long division to find the quotient & remainder:

$$(2x^4 - x^3 - 2) \div (2x^2 + x + 1)$$

dividend divisor

Fraction Form: $\boxed{x^2 - x + \frac{x-2}{2x^2+x+1}}$

$$
\begin{array}{r|l}
 & x^2 - x \\
2x^2+x+1 & 2x^4 - x^3 + 0x^2 + 0x - 2 \\
 & -(2x^4 + x^3 + x^2) \\
 & \quad -2x^3 - x^2 + 0x \\
 & \quad -(-2x^3 - x^2 - x) \\
 & \qquad x - 2
\end{array}
$$

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$f(x) = x^3 + 4x^2 + 7x - 9 \qquad d(x) = x + 3$

$$x^2 + x + 4 - \frac{21}{x+3}$$

$$
\begin{array}{r|l}
x+3 & x^3 + 4x^2 + 7x - 9 \\
 & -(x^3 + 3x^2) \\ \hline
 & \quad x^2 + 7x \\
 & -(x^2 + 3x) \\ \hline
 & \quad 4x - 9 \\
 & -(4x + 12) \\ \hline
 & \quad -21
\end{array}
$$

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Remainder Thm:

If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r = f(k)$

Find the remainder when $f(x) = 3x^2 + 7x - 20$ is divided by: $x - k$

a. $x-2$

$k=2$

$r = f(2)$

$= 3(2)^2 + 7(2) - 20$

$12 + 14 - 20$

$= 6$

b. $x+1$

$k=-1$

$r = f(-1)$

$= 3(-1)^2 + 7(-1) - 20$

$= 3 - 7 - 20$

$= -24$

c. $x+4$ factor

$k = -4$

$r = f(-4)$

$= 3(-4)^2 + 7(-4) - 20$

$= 48 - 28 - 20$

$= 0$

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Factor Thm:

A polynomial fnc $f(x)$ has a factor $x-k$ if $\underbrace{f(k)}_{\text{remainder}}=0$.

Is $\underset{k=-2}{x+2}$ a factor of $f(x)=x^3-3x+4$?

$r=f(-2)=(-2)^3-3(-2)+4$

$=-8+6+4$

$=2\neq 0$

No!

Oct 8-9:59 AM

Synthetic Division:

can only use when dividing by $\underline{x-k}$.

Divide: $(2x^3-3x^2-5x-12)\div(\underset{x-k}{x-3})$

$k=3$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & \downarrow & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & 0 \end{array}$$

$2x^2+3x+4$ remainder: 0

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$$(x^3+4x^2-6) \div (x+1)$$

$k=-1$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 0 & -6 \\ & \downarrow & -1 & -3 & 3 \\ \hline & 1 & 3 & -3 & -3 \end{array}$$

$$x^2+3x-3+\frac{-3}{x+1}$$

Oct 8-10:08 AM