2.5 Complex Zeros & the Fundamental Thm of Algebra

Obj: 1. Factor polynomials w/ real coefficients using factors with complex wefficients.

Fundamental Thm of Algebra:
a polynomial fnc of degree n has n complex zeros (real or nonreal)
* Some zeros can be repeated.

zeros, solutions, roots, x-intercepts

imaginary zero: (not x-intercept)

If k is a complex #: and

1. $x = k$ is a solution of $f(x) = 0$
2. k is a zero of $f(x)$
3. $x - k$ is a factor of $f(x)$.

Write as a polynomial in standard form.

Identify zeros & x-intercepts.
real or imag. real

$f(x) = (x-2i)(x+2i)$

$= x^2 + 2ix - 2ix - 4i^2$ ($i^2 = -1$)

$= x^2 + 4$

Zeros: $x-2i=0$ $x+2i=0$

$x=2i$ $x=-2i$

x intercepts: none

$f(x) = (x-5)(x-\sqrt{2}i)(x+\sqrt{2}i)$

Zeros: $x-5=0$ $x-\sqrt{2}i=0$ $x+\sqrt{2}i=0$

$*x=5$ $x=\sqrt{2}i$ $x=-\sqrt{2}i$

x-intercepts: $x=5$

$f(x) = (x-5)(x^2+\sqrt{2}ix-\sqrt{2}ix-2i^2)$ $+2$

$(x-5)(x^2+2)$

$= x^3-5x^2+2x-10$

$f(x) = (x-3)(x-3)(x-i)(x+i)$

Zeros: $x = 3, i, -i$

x intercepts: $x = 3$

Standard form:

$f(x) = (x^2-6x+9)(x^2+1)$

$= x^4+x^2-6x^3-6x+9x^2+9$

$= x^4-6x^3+10x^2-6x+9$

Complex Conjugate Zeros:

If a+bi is a zero, then a-bi is a zero.

Find a polynomial of minimal degree in standard form w/ zeros: -3, 4, 2-i, 2+i

$(x+3)(x-4)(x-(2-i))(x-(2+i))$

$(x+3)(x-4)(x-2+i)(x-2-i)$

$(x^2-x-12)(x^2-2x-ix-2x+4+2i+ix-2i-i^2)$

$i^2=-1$, $+1$

$(x^2-x-12)(x^2-4x+5)$

$x^4-4x^3+5x^2-x^3+4x^2-5x-12x^2+48x-60$

$x^4-5x^3-3x^2+43x-60$

Finding Complex Zeros:

1-2i is a zero of $f(x) = 4x^4 + 17x^2 + 14x + 65$.

1+2i

4 zeros

Find the remaining zeros.

$(x-1+2i)(x-1-2i)$

$x^2 - x - 2ix - x + 1 + 2i + 2ix - 2i + 4 - 4i^2$

$(x^2 - 2x + 5)(\quad) = f(x)$

$4x^2 + 8x + 13$

$x^2-2x+5 \overline{)\,4x^4 + 0x^3 + 17x^2 + 14x + 65}$

$-(4x^4 - 8x^3 + 20x^2)$

$8x^3 - 3x^2 + 14x$

$-(8x^3 - 16x^2 + 40x)$

$13x^2 - 26x + 65$

$-(13x^2 - 26x + 65)$

0

$4x^2 + 8x + 13$

$x = \dfrac{-8 \pm \sqrt{64 - 4(4)(13)}}{8} = \dfrac{-8 \pm \sqrt{-144}}{8} = \dfrac{-8 \pm 12i}{8}$

$x = \dfrac{-2 \pm 3i}{2}$