

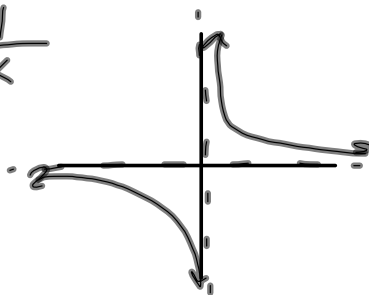
## 2.6 Graphs of Rational Functions

- Obj: 1. Describe graphs of rational functions.
2. Identify horizontal & vertical asymptotes.
3. Predict end behavior.

$$r(x) = \frac{f(x)}{g(x)}$$

Oct 25-9:11 AM

$$f(x) = \frac{1}{x}$$



$g(x) = \frac{1}{x-2}$  Find domain  
and use limits to describe end behavior

$$D: x-2 \neq 0$$

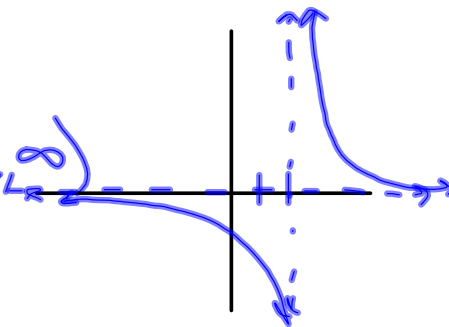
$$x \neq 2$$

$$(-\infty, 2) \cup (2, \infty)$$

\* end behavior

$$\lim_{x \rightarrow \infty} g(x) = 0$$

$$\lim_{x \rightarrow -\infty} g(x) = 0$$



Oct 25-9:22 AM

$$f(x) = \frac{-3}{x-1}$$

D:  $x \neq 1$   
 $(-\infty, 1) \cup (1, \infty)$

$\lim_{x \rightarrow \infty} f(x) = 0$   
 $\lim_{x \rightarrow -\infty} f(x) = 0$

Oct 25-9:28 AM

$$g(x) = \frac{2x^0}{x-3} = 2\left(\frac{1}{x-3}\right)$$

Describe the transformations,  
 find domain, identify asymptotes  
 graph by hand, end behavior.

v. str. by 2  
 right 3

D:  $x \neq 3$   
 $(-\infty, 3) \cup (3, \infty)$

VA:  $x = 3$   
 HA:  $y = 0$

$\lim_{x \rightarrow \infty} f(x) = 0$   
 $\lim_{x \rightarrow -\infty} f(x) = 0$

Oct 25-9:32 AM

$$h(x) = \frac{3x-7}{x-2} = \frac{-1}{x-2} + 3$$

$$x-2 \overline{) 3x-7}$$

$$\underline{-(3x-6)}$$

$$-1$$

refl. over x axis  
 right 2  
 up 3

VA:  $x=2$   
 HA:  $y=3$

$\lim_{x \rightarrow \infty} h(x) = 3$   
 $\lim_{x \rightarrow -\infty} h(x) = 3$

Oct 25-9:39 AM

$$f(x) = \frac{3x-2}{x-1} = 3 + \frac{1}{x-1}$$

$$x-1 \overline{) 3x-2}$$

$$\underline{-(3x-3)}$$

$$1$$

up 3  
 right 1

HA:  $y=3$   
 VA:  $x=1$

$D: x \neq 1$   
 $(-\infty, 1) \cup (1, \infty)$

$\lim_{x \rightarrow \infty} f(x) = 3$   
 $\lim_{x \rightarrow -\infty} f(x) = 3$

$\lim_{x \rightarrow 1^-} f(x) = -\infty$  (from left)  
 $\lim_{x \rightarrow 1^+} f(x) = \infty$  (from right)

Oct 25-9:44 AM

$$f(x) = \frac{-2}{x+5}$$

refl. over x-axis  
 v. str. by 2  
 left 5

VA:  $x = -5$   
 HA:  $y = 0$

D:  $x \neq -5$   
 $(-\infty, -5) \cup (-5, \infty)$

$\lim_{x \rightarrow \infty} f(x) = 0$   
 $\lim_{x \rightarrow -\infty} f(x) = 0$   
 $\lim_{x \rightarrow -5^-} f(x) = \infty$   
 $\lim_{x \rightarrow -5^+} f(x) = -\infty$

Oct 25-9:53 AM

$$f(x) = \frac{2x^2 - 1}{x^2 + 3} = 2 - \frac{7}{x^2 + 3}$$

$$\begin{array}{r} x^2 + 3 \overline{) 2x^2 + 0x - 1} \\ \underline{-(2x^2 \quad + 6)} \phantom{-1} \\ \phantom{x^2 + 3} -7 \phantom{-1} \end{array}$$

D:  $x^2 + 3 \neq 0$   
 $\sqrt{x^2} = \sqrt{-3}$   
 $x \neq \text{imag.}$   
 $(-\infty, \infty)$

HA:  $y = 2$   
 $\lim_{x \rightarrow \infty} f(x) = 2$   
 $\lim_{x \rightarrow -\infty} f(x) = 2$

Oct 25-10:00 AM

Find all asymptotes & intercepts:  
 $f(x) = \frac{x}{x^2-9}$

VA:  $x^2-9=0$     HA: none  
 $x^2=9$   
 $x=\pm 3$

\* Slant or Oblique Asymptote  $\frac{9x}{x^2-9}$   
 $x^2-9 \overline{) x^3 + 0x^2 + 0x + 0}$   
 $\underline{-(x^3 - 9x)} \quad \downarrow$   
 $9x + 0$

slant  $y=x$

x	y
-4	
4	
-1	$\frac{(-1)^3}{(-1)^2-9} = \frac{-1}{-8} = \frac{1}{8}$
0	$\frac{0}{0-9} = 0$
1	$\frac{1}{1-9} = -\frac{1}{8}$

Oct 25-10:04 AM

$g(x) = \frac{x^2+2}{x^2+2x-3}$  Find asymptotes, graph by hand.

VA:  $x^2+2x-3=0$     HA:  $y=0$   
 $(x+3)(x-1)=0$   
 $x=-3$      $x=1$

x	y
-5	$\frac{-5+2}{25-10-3} = \frac{-3}{12} = -\frac{1}{4}$
0	$\frac{2}{-3}$
-1	$\frac{1}{1-2-3} = -\frac{1}{4}$
-2	$\frac{0}{\dots} = 0$

Oct 25-10:11 AM

$$f(x) = \frac{2x^2 - 2}{x^2 - 4}$$

$$\text{VA: } x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\text{HA: } y = 2$$

x	y
-3	$\frac{2(9) - 2}{9 - 4} = \frac{16}{5} = 3\frac{1}{5}$
3	$\frac{16}{9 - 4} = \frac{16}{5}$
0	$\frac{-2}{-4} = \frac{1}{2}$
1	$\frac{2 \cdot 1 - 2}{1 - 4} = \frac{0}{-3}$
-1	0

Oct 25-10:21 AM

Find the end behavior asymptote:

$$f(x) = \frac{x^3 - 3x^2 + 3x + 1}{x - 1}$$

$$\begin{array}{r} \downarrow \quad \downarrow \quad -3 \quad 3 \quad 1 \\ \phantom{0} \quad \phantom{0} \quad 1 \quad -2 \quad 1 \\ \hline \text{EBA} \quad | \quad 1x^2 \quad -2x \quad +1 \quad + \quad \frac{2}{x-1} \end{array}$$

Oct 25-10:29 AM