

## 2.8 Solving Inequalities in One Variable

Obj: 1. Solve ineq. with polynomials & rational fns algebraically & graphically.

$$f(x) = (x+3)(x^2+1)(x-4)^2$$

Find the zeros & the values that are positive & negative.

Sign chart :  $x+3=0$

$$x = -3$$

~~$x^2+1=0$~~   
 ~~$\sqrt{x^2+1} = \sqrt{-1}$~~   
 ~~$x = \pm i$~~

$$x-4=0$$

$$x = 4$$

(-)	(+)	(-) <sup>2</sup>	(+)	(+)	(-) <sup>2</sup>	(+)	(+)	(+) <sup>2</sup>
-4	-3	0	4	5	5	5	5	5
neg.			POS			POS		

Zeros: -3, 4

POS:  $(-3, 4) \cup (4, \infty)$

neg:  $(-\infty, -3)$

$$f(x) = (x-7)(3x+1)(x+4)$$

$$\text{Zeros: } x = -4, -\frac{1}{3}, 7$$

(-)(-)(-)	(-)(-)(+)	(-)(+)(+)	(+)(+)(+)
neg	neg	neg	pos
-5	-2	6	8
-4	- $\frac{1}{3}$	7	8
	pos		

$$\text{POS: } (-4, -\frac{1}{3}) \cup (7, \infty)$$

$$\text{NEG: } (-\infty, -4) \cup (-\frac{1}{3}, 7)$$

Solve analytically:

$$2x^3 - 7x^2 - 10x + 24 > 0.$$

$$\frac{P}{Q} = \frac{\pm 1, 2, 3, 4, 6, 8, 12, 24}{\pm 1, 2}$$

looks like:

$$(4)(-2)\left(\frac{3}{2}\right)$$

$$2(x-4)(x+2)\left(x-\frac{3}{2}\right)$$

$$\begin{array}{r} 4 \overline{) 2 \quad -7 \quad -10 \quad 24} \\ \underline{\phantom{4} \downarrow \phantom{0} \phantom{0} \phantom{0}} \phantom{0} \phantom{0} \phantom{0} \phantom{0}} \\ 2 \quad -7 \quad -10 \quad 24 \\ \underline{\phantom{0} \phantom{0} \phantom{0} \phantom{0}} \phantom{0} \phantom{0} \phantom{0} \phantom{0}} \\ 8 \quad 4 \quad -24 \\ \underline{\phantom{0} \phantom{0} \phantom{0} \phantom{0}} \phantom{0} \phantom{0} \phantom{0} \phantom{0}} \\ 0 \quad 0 \quad 0 \end{array}$$

$$\begin{array}{r} -2 \overline{) 2 \quad -1 \quad -6 \quad 0^*} \\ \underline{\phantom{-2} \phantom{0} \phantom{0} \phantom{0}} \phantom{0} \phantom{0} \phantom{0}} \\ 2 \quad -1 \quad -6 \quad 0^* \\ \underline{\phantom{0} \phantom{0} \phantom{0} \phantom{0}} \phantom{0} \phantom{0} \phantom{0}} \\ 0 \quad 0 \quad 0 \end{array}$$

$$\begin{array}{r} \frac{3}{2} \overline{) 2 \quad -3 \quad 0^*} \\ \underline{\phantom{\frac{3}{2}} \phantom{0} \phantom{0} \phantom{0}} \phantom{0} \phantom{0} \phantom{0}} \\ 2 \quad -3 \quad 0^* \\ \underline{\phantom{0} \phantom{0} \phantom{0} \phantom{0}} \phantom{0} \phantom{0} \phantom{0}} \\ 0 \quad 0 \quad 0 \end{array}$$

(-)(-)(-) (-)(+)(-) (-)(+)(+)

neg | | | neg | | | pos

-3 | -2 | 0 | 3 | 2 | 4 | 5

pos

$$\left(-2, \frac{3}{2}\right) \cup (4, \infty)$$

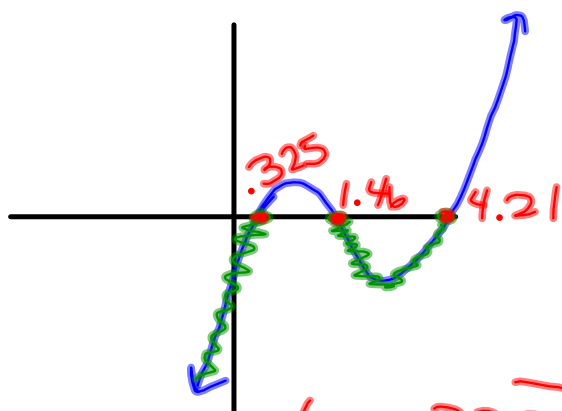
Solve Graphically:

$$x^3 - 6x^2 \leq 2 - 8x$$

$$\begin{array}{r} -2 + 8x \end{array} \quad \begin{array}{r} -2 + 8x \end{array}$$

$$x^3 - 6x^2 + 8x - 2 \leq 0$$

Graph  $y = x^3 - 6x^2 + 8x - 2$



$$(-\infty, .325] \cup [1.46, 4.21]$$

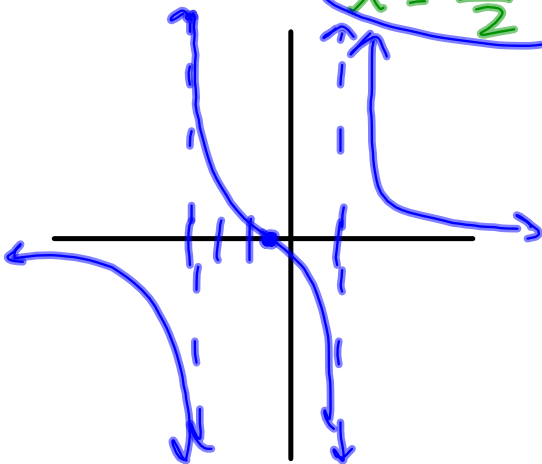
$$r(x) = \frac{(2x+1)}{(x+3)(x-1)}$$

Sign chart: Find values that are zeros & undef. <sup>VA</sup>

VA:  $(x+3)=0$   $x-1=0$   
 $x=-3$   $x=1$

Zeros:  $2x+1=0$   
 $x=-\frac{1}{2}$

Sign chart:  
 $\frac{(-)}{(-)(-)} \text{ POS } | \text{ neg } | \text{ POS.}$   
 neg.  $-3$   $-\frac{1}{2}$   $1$



$$f(x) = \frac{x-1}{(2x+3)(x-4)}$$

$$\text{VA: } (2x+3)(x-4) = 0$$
$$x = -\frac{3}{2}, 4$$

$$\text{Zeros: } x-1 = 0$$
$$x = 1$$

neg | pos | neg | pos

$-\frac{3}{2}$  | 1 | 4

