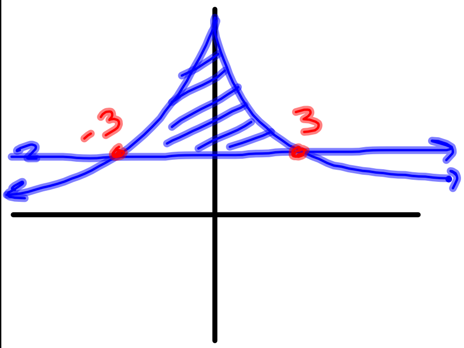
1. $y=\frac{20}{1+x^2}$ $\quad y=2$

bounds: 1 pt.



a. $A=\int_{-3}^{3}\left(\underbrace{\frac{20}{1+x^2}-2}_{1}\right)dx$

or

or

$2\int_0^3\left(\frac{20}{1+x^2}-2\right)dx$

≈ 37.962 1

b. $V=2\pi\int_0^3\underbrace{\left[\left(\frac{20}{1+x^2}\right)^2-2^2\right]}_{2}dx \approx \underbrace{1871.190}_{1}$

c. $A=\frac{1}{2}\pi r^2$

$V=\cancel{2\left(\tfrac{1}{2}\pi\right)}\int_0^3\left(\frac{1}{2}\left(\frac{20}{1+x^2}-2\right)\right)^2 dx$

$\pi\int_0^3\underbrace{\left[\frac{1}{4}\left(\frac{20}{1+x^2}-2\right)^2\right]}_{2}dx \approx \underbrace{174.268}_{1}$

2. a. $\int_0^7 f(t)\,dt \approx 8264$ gal.
1 1

b. where $f(t) < g(t)$ 1

1 $(0, 1.617) \cup (3, 5.076)$

c. possible: 0, 7, 3

1 $t=3$ when $f(t)-g(t)$ changes sign

* $t=0$ 5000 gal.

* $t=3$ $5000 + \int_0^3 f(t)\,dt - 250(3)$ 1

1 5127 gal. ≈ 5126.591 gal.
1

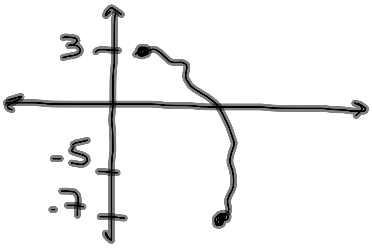
* $t=7$ $5126.591 + \int_3^7 f(t)\,dt - 2000(4)$

1 ≈ 4513.807 gal.

3. $h(x) = f(g(x)) - 6$

a. 1 $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$ $(1,3)$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$ $(3,-7)$



Since h is cont and $h(3) < -5 < h(1)$ the

1 IVT says there exists an r value $1 < r < 3$ such that

$h(c) = -5$

b. 1 $\dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{3 - 1} = -5$

Since h is cont & diff. the MVT says 1 there exists a c value $1 < c < 3$ such that $h'(c) = -5$

c. $w(x) = \int_1^{g(x)} f(t)\,dt$ Fundamental Thm. of Calculus

$w'(x) = f(g(x)) \cdot g'(x)$

$w'(3) = f(g(3)) \cdot g'(3) = -2$ 1

d. $y = m(x - x_1) + y_1$ $x = 2$ $x_1 = 2$, $y_1 = 1$

$g^{-1}(2) = 1$ 1

Section 3.8

$g(a) = b \rightarrow g^{-1}(b) = a$

$g'(a) = c \longrightarrow (g^{-1})'(b) = \frac{1}{c}$

$g'(1) = 5 \rightarrow (g^{-1})'(2) = \frac{1}{5}$ 1

$y = \frac{1}{5}(x - 2) + 1$ 1

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4. $x(t) = e^{-t}\sin t \quad [0, 2\pi]$

a. possible: $0, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}$

$x'(t) = -e^{-t}\sin t + e^{-t}\cos t$ 2

$e^{-t}(-\sin t + \cos t) = 0$ 1

$e^{-t} \neq 0 \quad -\sin t + \cos t = 0$

$\sin t = \cos t$

$t = \frac{\pi}{4}, \frac{5\pi}{4}$

$* \; x(0) = e^{0} \cdot \sin 0 = 0$

$* \; x(\frac{\pi}{4}) = e^{-\frac{\pi}{4}} \cdot \sin\frac{\pi}{4} > 0$

$* \; x(\frac{5\pi}{4}) = e^{-\frac{5\pi}{4}} \cdot \sin\frac{5\pi}{4} < 0$

justify

1 $x(2\pi) = e^{-2\pi} \cdot \sin 2\pi = 0$

1 $t = \frac{5\pi}{4}$ → only neg pos. ✓

b. $A x''(t) + x'(t) + x(t) = 0$

$x'(t) = e^{-t}(-\sin t + \cos t)$

$x''(t) = -e^{-t}(-\sin t + \cos t) + e^{-t}(-\cos t - \sin t)$

2 $= \cancel{e^{-t}\sin t} - e^{-t}\cos t - e^{-t}\cos t - \cancel{e^{-t}\sin t}$

$= -2e^{-t}\cos t$

1 $A(-2e^{-t}\cos t) - \cancel{e^{-t}\sin t} + e^{-t}\cos t + \cancel{e^{-t}\sin t} = 0$

$-2Ae^{-t}\cos t + e^{-t}\cos t = 0$

$e^{-t}\cos t(-2A + 1) = 0$

$-2A + 1 = 0$

$A = \frac{1}{2}$ 1

5. r is concave down: r'' is neg
r' is dec.

$r = 30 \quad t = 5$

$V = \frac{4}{3}\pi r^3$

a. $r(5.4) \approx r(5) + r'(5)\Delta t$

$30 + 2(.4) = 30.8$ ft. 1 pt.

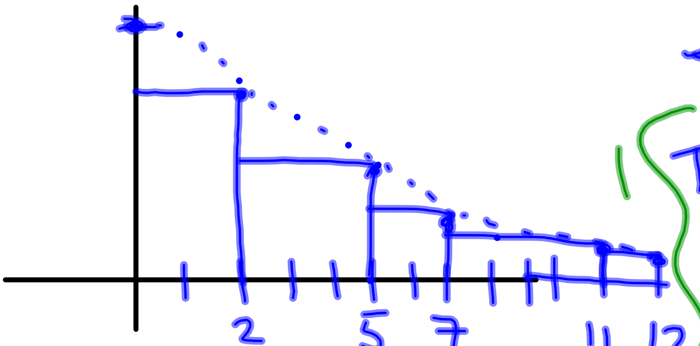
1pt. *greater - b/c $r(t)$ is c.down

b. 2 $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(30)^2(2)$

$= 7200\pi$ ft^3/min 1

c. $\int_0^{12} r'(t)\,dt \approx 2(4) + 3(2) + 2(1.2) + 4(.6) + 1(.5)$

≈ 19.3 ft 1



1 The amount of feet that the rad. changes from $t=0$ to $t=12$.

d. 1 less than - b/c $r(t)$ is concave down from $[0, 12]$ and when $r(t)$ is c. down RRAM is a lesser approx. 1

6. $f(x) = k\sqrt{x} - \ln x$

a. $f'(x) = \dfrac{k}{2\sqrt{x}} - \dfrac{1}{x}$ 1

$f''(x) = \dfrac{k}{2}\left(-\dfrac{1}{2}x^{-\frac{3}{2}}\right) + \dfrac{1}{x^2}$ 1

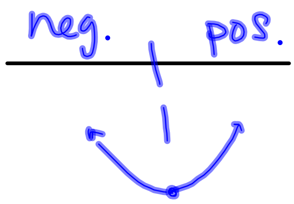
$= \dfrac{-k}{4x^{\frac{3}{2}}} + \dfrac{1}{x^2}$

b. CP → when $f'(x) = 0$ or

1 $f'(1) = 0$

$\dfrac{k}{2} - 1 = 0$

1 $k = 2$



1 minimum: 1 Since f' is neg from $(0,1)$, f is dec. and f' is pos $(1,\infty)$, so f is inc.

c. IP → 1 $f''(x) = 0$ or $f(x) = 0$

$\dfrac{-k}{4x^{\frac{3}{2}}} + \dfrac{1}{x^2} = 0$

$\dfrac{1}{x^2} = \dfrac{k}{4x^{\frac{3}{2}}}$

$kx^2 = 4x^{\frac{3}{2}}$

$k = \dfrac{4x^{\frac{3}{2}}}{x^2} = \dfrac{4}{\sqrt{x}}$

$k\sqrt{x} - \ln x = 0$

$k\sqrt{x} = \ln x$

$k = \dfrac{\ln x}{\sqrt{x}}$

1 $\dfrac{4}{\cancel{\sqrt{x}}} = \dfrac{\ln x}{\cancel{\sqrt{x}}}$

$4 = \ln x$

$e^4 = x$

$k = \dfrac{4}{\sqrt{e^4}} = \dfrac{4}{e^2}$ 1