2008 FR

1. a. $A = \underset{\text{1pt.}}{\underline{\int_0}}^{2} \left(\underset{\text{1pt.}}{\underline{\sin\pi x - (x^3-4x)}}\right)dx \ = \underset{\text{1pt}}{\underline{4}}$

b. $A = \underset{\text{1 pt.}}{\underline{\int_{.539}^{1.675}}} \left(\underset{\text{1 pt.}}{\underline{-2-(x^3-4x)}}\right)dx$

c. $A = s^2 = \left(\sin\pi x - (x^3-4x)\right)^2$

$V = \int_0^2 \left(\underset{\text{1 pt.}}{\underline{\sin\pi x - (x^3-4x)}}\right)^2 dx \approx \underset{\text{1 pt.}}{\underline{9.978}}$

d. $h(x) = 3-x$

$V = \int_0^2 \underset{\text{1 pt.}}{\underline{(3-x)\left(\sin\pi x - (x^3-4x)\right)}}dx$

$\approx \underset{\text{1 pt.}}{\underline{8.370}}$

2. a. $L'(5.5) \approx \frac{L(7)-L(4)}{7-4} = \underbrace{8}_{1pt.}$ $\underbrace{\text{people/hr.}}_{1\text{ pt.}}$

b. $\frac{1}{2}(120+156)+\frac{2}{2}(156+176)+\frac{1}{2}(176+126)$

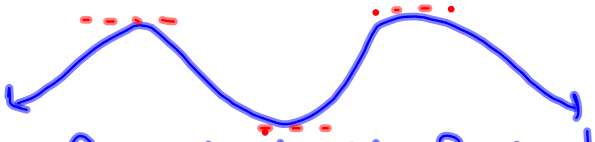
$= \frac{\boxed{621}}{4\text{ hrs.}}$ 1pt. $= \underbrace{155.25 \text{ people}}_{1pt.}$

c.

0	1	3	4	7	8	9
120	156	176	126	150	80	0

inc dec inc dec

1pt. →



$L'(t)=0$ for at least $\underbrace{\text{3 values}}_{1pt.}$ $[0,9]$

b/c $\underbrace{\text{L(t) changes from inc to dec or}}_{1pt.}$

dec to inc at least 3 times.

d. $\int_0^{13} \underbrace{r(t)\,dt}_{1pt.} \approx \underbrace{973}_{1pt.}$ tickets

3. a. 1 pt. $\left\{ \frac{dV}{dt}=2000 \quad \frac{dr}{dt}=2.5 \right.$

$V=\pi r^2 h$

$\frac{dh}{dt}=?$

$r=100 \quad h=.5$

2 pt. $\underline{\frac{dV}{dt}=\pi\left(2r\frac{dr}{dt}h+r^2\boxed{\frac{dh}{dt}}\right)}$

$2000=\pi\left(200(2.5)(.5)+100^2\frac{dh}{dt}\right)$

$\frac{dh}{dt}\approx \underline{.039}$ 1 pt

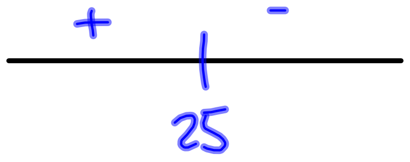
b. $\frac{dV}{dt}=0$

$\frac{dV}{dt}=2000-R(t)=0$

$\underline{R(t)=2000}$ 1pt.

$400\sqrt{t}=2000$

$\underline{t=25}$ min. 1pt.



1pt. $\Bigg\{$ @ $t=25$, the volume reaches a max since $\frac{dV}{dt}>0$ before $t=25$ and $\frac{dV}{dt}<0$ after 25 min.

c. $\underbrace{60000}_{\text{1pt.}}+\int_0^{25}\underbrace{(2000-R(t))}_{\text{1pt.}}dt$

4. a. $v(t)<0 \quad (0,3)\cup(5,6)$

$v(t)>0 \quad (3,5)$

possible: $\underset{1\text{pt.}}{\underline{t=3,\ 6}}$

$x(3) = x(0) + \int_0^3 \underset{1\text{pt.}}{\underline{v(t)dt}} = -2-8 = -10$

$x(6) = x(0) + \int_0^6 v(t)dt = -2-8+3-2 = -9$

$\underset{1\text{ pt.}}{\underline{@\,t=3,\ x(t)=-10}}$, so it is farther left.

b. $x(0)=-2$ $\big\}\,-8$

$1\text{pt.}\left\{\begin{array}{l} x(3)=-10 \;\big\}\,-8 \\ x(5)=-7 \;\big\}\,-8 \\ x(6)=-9 \end{array}\right.$

since $x(t)$ is $\underline{\text{cont}}$, 1pt

by IVT, $x(t)=-8$

$\underset{1\text{pt.}}{\underline{3 \text{ times}}}$.

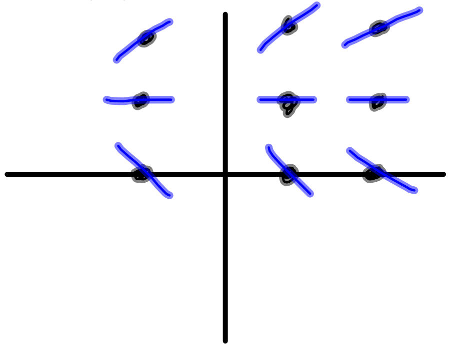
c. the particle's speed is $\underline{\text{dec.}}$ b/c

$\underline{v(t)<0 \text{ and } a(t)>0}$ 1pt.

d. $\underset{1\text{pt.}}{\underline{(0,1)\cup(4,6)}} \rightarrow \underset{1\text{pt.}}{\underline{v \text{ is dec.}}}$

5. $\frac{dy}{dx} = \frac{y-1}{x^2} \quad x \neq 0$

a.



zero slopes: 1pt.

all others: 1pt.

b. $\frac{dy}{y-1} = \frac{dx}{x^2}$ 1 pt.

2pts → $\ln|y-1| = -\frac{1}{x} \boxed{+C}$ 1 pt. $f(2)=0$

$\ln|0-1| = -\frac{1}{2}+C$

$0 = -\frac{1}{2}+C$

$\underline{C=\frac{1}{2}}$ 1 pt.

$\ln|y-1| = -\frac{1}{x}+\frac{1}{2}$

$e^{-\frac{1}{x}+\frac{1}{2}} = y-1$

$\underline{y = 1+e^{-\frac{1}{x}+\frac{1}{2}}}$ 1pt.

c. $\lim_{x\to\boxed{\infty}}\left(1+e^{-\frac{1}{x}+\frac{1}{2}}\right)$

$= \underline{1+e^{\frac{1}{2}}}$ 1pt.

6. $x = e^2$

a. $y = m(x - x_1) + y_1$

$m = f'(e^2) = \frac{1 - \ln e^2}{e^4} = \frac{-1}{e^4}$

$f(e^2) = \frac{\ln e^2}{e^2} = \frac{2}{e^2}$ $\Big\}$ 1 pt.

$y = \underline{\frac{-1}{e^4}(x - e^2) + \frac{2}{e^2}}$ 1 pt.

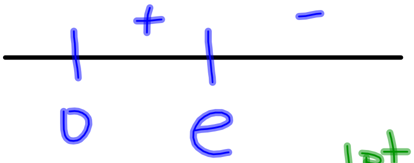
b. $f'(x) = 0$

$\frac{1 - \ln x}{x^2} = 0$

$1 - \ln x = 0$

$\ln x = 1$

$\underline{x = e}$ 1pt.



there is a $\underline{\text{max @}}$ $x = e$ 1pt.

since $\underline{\text{f' changes from pos to}}$ neg. 1pt.

c. $f''(x) = 0$

$f''(x) = \frac{x^2(-\frac{1}{x}) - (1 - \ln x)(2x)}{x^4}$

2pts. $\nearrow$

$\frac{-3 + 2\ln x}{x^3} = 0$

$-3 + 2\ln x = 0$

$\ln x = \frac{3}{2}$

$\underline{x = e^{\frac{3}{2}}}$ 1 pt.

d. $\lim_{x \to 0^+} \frac{\ln x}{x}$ $x \neq 0$ V.A

$= \underline{-\infty \text{ or DNE}}$

1 pt.