2009 FR

1\. a. $a(t) = v'(t) = \text{slope}$

slope @ $t = 7.5$

$$a(7.5) = \frac{.2 - .3}{8 - 7} = \underbrace{-.1}_{1} \ \underbrace{mi/min^2}_{1}$$

b. $\int_0^{12} |v(t)|\,dt$ is the $\underbrace{\text{total distance traveled}}_{1}$ from $t=0$ to $t=12$.

$$\int_0^{12} |v(t)|\,dt = \int_0^2 v(t)\,dt + \left|\int_2^4 v(t)\,dt\right| + \int_4^{12} v(t)\,dt$$

$$= .2 + |-.2| + 1.4 = \underline{1.8 \text{ miles}} \quad 1$$

c. @ $\underbrace{t = 2 \text{ min}}_{1}$ — Her $\underbrace{\text{velocity changes}}_{1}$ from pos to neg.

d. $\int_0^{12} w(t)\,dt = \underline{1.6 \text{ mi}}$ 2

$$\int_0^{12} v(t)\,dt = .2 - .2 + 1.4 = 1.4 \text{ mi}$$

$\underbrace{\text{Caren lives closer}}_{1}$

2. a. $\underline{\int_0^2 R(t)\,dt} = \underline{980 \text{ people}}$

b. $\underline{R'(t) = 0}$

$R'(t) = 2760t - 2025t^2 = 0$

$\underline{t = 1.36296}$

Possible max: $t = 0$ $\quad t = 1.36296$ $\quad t = 2$

$R(0) = 0$

$R(1.36296) = 854.527$ people

$R(2) = 120$ people

The max rate occurs @ $\underline{t = 1.363 \text{ hrs}}$

c. $w'(t) = (2-t)R(t)$

$w(2) - w(1) = \underline{\int_1^2 w'(t)\,dt} = \int_1^2 (2-t)(1380t^2 - 675t^3)\,dt$

$\approx \underline{387.5 \text{ hrs.}}$

d. $\underline{\frac{1}{980}\int_0^2 (2-t)R(t)\,dt} \approx \underline{.77551 \text{ hrs.}}$

3. a. Profit: $R(x) - C(x)$

$$= 120 \cdot 25 - \underline{\int_0^{25} 6\sqrt{x}\,dx}$$

$$= \underline{2500 \text{ dollars}}$$

b. $\int_{25}^{30} 6\sqrt{x}\,dx$ is the cost to produce a section of cable from 25 to 30 m.

c. Profit : $\underline{120 \cdot K} - \int_0^{K} \underline{6\sqrt{x}\,dx}$

$$\int_0^{K} (120 - 6\sqrt{x})\,dx$$

d. $\underline{P' = 0}$ $\quad P'(x) = 120 - 6\sqrt{x}$

$$P'(K) = 120 - 6\sqrt{K} = 0$$

$$\underline{K = 400}$$

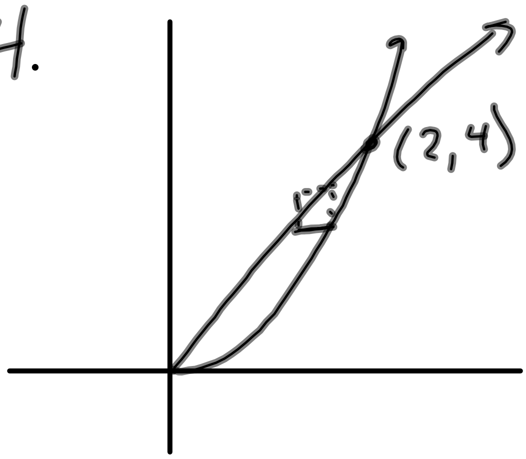
pos | neg

400

Since $K = 400$ is the only crit pt & $\underline{P' \text{ changes}}$ $\underline{\text{from pos to neg}}$, $K = 400$ is a max.

$$P(400) = \underline{\$16000}$$

4.



a. $A = \int_0^2 (2x - x^2)\,dx$ 1

$= \left(x^2 - \frac{1}{3}x^3\right)\Big|_0^2 = 4 - \frac{8}{3} - 0 = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$ 1 1

b. $A(x) = \sin\left(\frac{\pi}{2}x\right)$.

$V = \int_0^2 \sin\left(\frac{\pi}{2}x\right)dx$ 1

$u = \frac{\pi}{2}x$
$du = \frac{\pi}{2}dx$
$\frac{2}{\pi}du = dx$

$\frac{2}{\pi}\int \sin u\,du = -\frac{2}{\pi}\cos u\Big| = -\frac{2}{\pi}\cos\frac{\pi}{2}x\Big|_0^2$ 1

$-\frac{2}{\pi}(\cos\pi - \cos 0) = -\frac{2}{\pi}(-1-1) = \frac{4}{\pi}$ 1

c. $A = s^2$

$x = \frac{1}{2}y \quad x = \sqrt{y}$

$V = \int_0^4 \left(\sqrt{y} - \frac{1}{2}y\right)^2 dy$ 1 2

5.

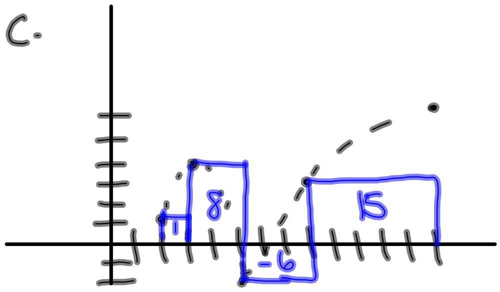
a. $f'(4) \approx \dfrac{f(5)-f(3)}{5-3} = \underline{-3}$ 1

b. $\displaystyle\int_2^{13}(3-5f'(x))\,dx = \int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$

$\underline{3x\Big|_2^{13} - 5f(x)\Big|_2^{13}}$ 1

$(39-6)-5(f(13)-f(2)) = 33-5(6-1) = \underline{8}$ 1

c.



LRAM: 1

$\displaystyle\int_2^{13} f(x)\,dx \approx \underline{18}$ 1

d. $f'(5)=3$

$f''(x)<0$ – concave down

tangent line: slope: $f'(5)=3$

$f(5)=-2$

$\underline{y+2=3(x-5)}$ 1

$y=3(x-5)-2$

Since $f''(x)<0$ on the interval $5 \le x \le 8$, the tangent line @ $x=5$ lies above the graph

so $\underline{f(7) \le 3(7-5)-2=4}$ 1

secant line: m: $\dfrac{f(8)-f(5)}{8-5} = \dfrac{5}{3}$ $(5,-2)$

$\underline{y+2=\frac{5}{3}(x-5)}$ 1

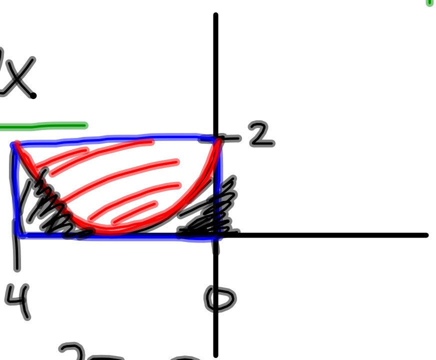
$y=\frac{5}{3}(x-5)-2$

$\underline{f(7) \ge \frac{5}{3}(7-5)-2 = \frac{4}{3}}$ 1

6.

a. f' changes from dec to inc @ $x=-2$
and from inc to dec @ $x=0$
so f has inflection points @ $x=-2$, $x=0$

b. $f(-4) = 5 + \int_0^{-4} g(x)\,dx$



$= 5 - \int_{-4}^{0} g(x)\,dx$

$5 - (\underbrace{8}_{\text{area of } \square} - \underbrace{2\pi}_{\text{area of semi-circ.}})$

$= 2\pi - 3$

$f(4) = 5 + \int_0^4 (5e^{(-x/3)} - 3)\,dx$

$5 + \left(-15e^{-\frac{x}{3}} - 3x\right)\Big|_0^4$

$5 + (-15e^{-\frac{4}{3}} - 12) - (-15) = 8 - 15e^{-\frac{4}{3}}$

c. $f'(x) = 0 \quad x = -2 \quad x = 3\ln\frac{5}{3}$

Since $f'(x)$ changes from pos to neg @ $x = 3\ln\frac{5}{3}$ that is max of f.