2010 FR

1. a. $\underbrace{\int_0^6 f(t)\,dt}_{1\text{pt.}} \approx \underbrace{142.275\ ft^3}_{1\text{pt.}}$

b. $f(8) - g(8) = \underbrace{-59.583}_{1\text{pt.}}\ ft^3/hr$

c.

$$h(t) = \begin{cases} h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds \\ h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds \\ h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds \end{cases}$$

$$\begin{array}{l} 1 \to \\ 1 \to \\ 1 \to \end{array} = \begin{cases} 0 & 0 \le t \le 6 \\ 125(t-6) & 6 < t \le 7 \\ 125 + 108(t-7) & 7 < t \le 9 \end{cases}$$

d. $\underbrace{\int_0^9 f(t)\,dt}_{1\text{pt.}} - \underbrace{h(9)}_{1\text{pt.}} \approx \underbrace{26.335}_{1\text{pt.}}\ ft^3$

2\. a. $E'(6) \approx \frac{E(7)-E(5)}{7-5} = \underline{4 \text{ hundred entries/hr.}}$ 1pt.

b. $\frac{1}{8}\int_0^8 E(t)\,dt = \frac{1}{8}\Big(\frac{2}{2}(4+0) + \frac{3}{2}(13+4)$
$+ \frac{2}{2}(21+13) + \frac{1}{2}(23+21)\Big)$ 1pt.

$\approx \underline{10.688}$ 1pt.

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average # of hundreds of entries in the box between noon & 8 pm. 1 pt.

c. $\underline{23 - \int_8^{12} P(t)\,dt}$ $= 23 - 16 = \underline{7 \text{ hundred entries}}$

1pt. 1pt.

d. $\underline{P'(t) = 0}$ $t = 9.1835, 10.8165$

1pt.

t	$P(t)$
8	0
9.1835	5.08866
10.8165	2.911
12	8

1pt.

Processed quickest @ $\underline{t = 12}$ 1pt.

3. a. $\underbrace{\int_0^3 r(t)\,dt}_{1\text{pt.}} = \frac{2}{2}(1200+1000) + \frac{1}{2}(1200+800)$

$= \underbrace{3200 \text{ people}}_{1\text{pt.}}$

b. $\underline{\text{inc}}$: 1 pt on $2 < t < 3$, $r(t) > 800$

c. $r(t) = 800$ $\quad \underbrace{t = 3}_{}$ 1pt.

$\underline{\text{For } 0 \le t < 3 \; r(t) > 800 \text{ and for}}$

$\underline{3 < t \le 8, \; r(t) < 800}$ $\therefore$ the line is longest 1pt.

@ $t = 3$.

$700 + 3200 - 800 \cdot 3 = \underbrace{1500 \text{ people}}_{1\text{pt.}}$

d. $\underbrace{0}_{1\text{pt.}} = 700 + \underbrace{\int_0^t r(s)\,ds}_{1\text{pt.}} - \underbrace{800t}_{1\text{pt.}}$

4. a. $\int_0^9 \underbrace{(6-2\sqrt{x})}_{\text{1pt.}}dx = \underbrace{\left(6x - \frac{4}{3}x^{\frac{3}{2}}\right)}_{\text{1pt.}}\Big|_0^9 = \underbrace{18}_{\text{1pt.}}$

b. $V = \underbrace{\pi\int_0^9}_{\text{1pt.}} \Big[\underbrace{(7-2\sqrt{x})^2 - (7-6)^2}_{\text{2 pts.}}\Big]dx$

c. $x = \frac{y^2}{4}$ $A = \left(\frac{y^2}{4}\right)\left(\frac{3y^2}{4}\right) = \frac{3y^4}{16}$

$V = \underbrace{\int_0^6}_{\text{1pt.}} \underbrace{\frac{3}{16}y^4}_{\text{2pts.}} dy$

5. a. $g(3) = \underline{g(0)} + \int_0^3 g'(x)\,dx$ 1pt.

$= 5 + \frac{\pi}{4}(2^2) + \frac{3}{2} = \underline{\frac{13}{2} + \pi}$ 1pt.

$g(-2) = 5 + \int_0^{-2} g'(x)\,dx = \underline{5 - \pi}$ 1pt.

b. $\underline{x = 0,\ x = 2,\ x = 3}$ 1pt.

b/c $\underline{g' \text{ changes}}$ from inc to dec or dec to inc. 1pt.

c. $h(x) = g(x) - \frac{1}{2}x^2$

1pt. $\underline{h'(x) = g'(x) - x} = 0$

$g'(x) = x$

$[-2, 2]$: $\sqrt{4 - x^2} = x$

1pt. $\underline{x = \sqrt{2} \quad \text{also} \quad x = 3}$

$h'(x) > 0 \quad [0, \sqrt{2})$

$h'(x) \le 0 \quad (\sqrt{2}, 5]$

$\underline{x = 3 \text{ is neither}}$ 1pt.

$\underline{x = \sqrt{2} \text{ is a max.}}$ 1pt.

6. a. $f'(1) = \left.\frac{dy}{dx}\right|_{(1,2)} = \underline{8}$ 1pt

$y = \underline{8(x-1)+2}$ 1pt.

b. $f(1.1) \approx \underline{2.8}$ 1pt.

since $y = f(x) > 0$ $\quad [1, 1.1)$

$\frac{d^2y}{dx^2} = y^3(1+3x^2y^2) > 0$

$\therefore$ $(1, 1.1)$ the line tangent to the graph of $y=f(x)$ @ $x=1$ lies below the curve and the approx $\underline{2.8 < f(1.1)}$
1pt.

c. $f(1)=2$

$\frac{dy}{dx} = xy^3$

1pt. $\frac{dy}{y^3} = x\,dx$

1pt. $-\frac{1}{2y^2} = \frac{1}{2}x^2 + C$ 1pt.

$-\frac{1}{2(2^2)} = \frac{1}{2}(1^2) + C$

$-\frac{1}{8} = \frac{1}{2} + C$

$\underline{C = -\frac{5}{8}}$ 1pt.

$-\frac{1}{2y^2} = \frac{1}{2}x^2 - \frac{5}{8}$

$\frac{1}{y^2} = -x^2 + \frac{5}{4}$

$1 = y^2\left(-x^2 + \frac{5}{4}\right)$

$y^2 = \frac{1}{-x^2 + \frac{5}{4}}$

1pt. $y = \sqrt{\frac{1}{-x^2 + \frac{5}{4}}}$

Apr 30-8:30 AM