2011 FR

1. a. $v(5.5) \approx -.453$
$a(5.5) \approx -1.359$

The speech is inc. @ $t = 5.5$ b/c
$v(5.5)$ & $a(5.5)$ have the same sign.
1pt. 1pt.

b. $\frac{1}{6}\int_0^6 v(t)\,dt \approx 1.949$
1pt. 1pt.

c. $\int_0^6 |v(t)|\,dt \approx 12.573$
1pt. 1pt.

d. $v(t) = 0$ when $t = 5.19552$
1pt.

$x(5.19552) = 2 + \int_0^{5.19552} v(t)\,dt \approx 14.135$
1pt. 1pt.

2. a. $H'(3.5) \approx \frac{H(5)-H(2)}{5-2} = \frac{52-60}{3} = -2.667°/min$ 1pt.

b. $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature in °C/min over the 10 min. 1pt.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(\frac{2}{2}(66+60)+\frac{3}{2}(52+60)+\frac{4}{2}(44+52)+\frac{1}{2}(44+43)\right)$$ 1pt.

$$\approx 52.95$$ 1pt.

c. $\int_0^{10} H'(t)\,dt = H(10)-H(0) = 43-66 = -23$ 1pt

The temp of tea drops 23° (from $t=0$ to $t=10$ 1pt.

d. $B(10) = 100 + \int_0^{10} B'(t)\,dt \approx 34.183$ 1pt. 1pt.

$H(10) - B(10) = 8.817$ 1pt.

The biscuits are 8.817°C cooler than the tea.

3. a. $f(x) = 8x^3$

$f'(x) = 24x^2$

$f'(\frac{1}{2}) = \underline{6}$ 1pt

$f(\frac{1}{2}) = 1$

$y = 6(x - \frac{1}{2}) + 1$ ← 1pt.

b. $A = \int_0^{\frac{1}{2}} \left(\underline{\sin(\pi x) - 8x^3}\right) dx$ 1pt.

$\left(\underline{-\frac{1}{\pi}\cos \pi x} - \underline{2x^4}\right)\Big|_0^{\frac{1}{2}} = -\frac{1}{\pi}\cancel{\cos\frac{\pi}{2}} - \frac{1}{8} - \left(-\frac{1}{\pi}\cos 0\right)$

1pt. 1pt.

$= -\frac{1}{8} + \frac{1}{\pi}$ ← 1

c. $V = \underline{\pi \int_0^{\frac{1}{2}}} \left[\underline{(1 - 8x^3)^2 - (1 - \sin(\pi x))^2} \right] dx$

1pt. 2pts.

4. a. $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = \underline{-6 - \frac{9\pi}{4}}$ 1pt.

$g'(x) = \underline{2 + f(x)}$ 1pt.

$g'(-3) = 2 + f(-3) = 2 + 0 = \underline{2}$ 1pt.

b. $\underline{g'(x) = 0}$ when $f(x) = -2$ @ $\underline{x = \frac{5}{2}}$
1pt. 1pt.

1pt. $\begin{cases} g'(x) > 0 & (-4, \frac{5}{2}) \\ g'(x) < 0 & (\frac{5}{2}, 3) \end{cases}$ $\therefore$ g has an abs. max @ $x = \frac{5}{2}$

c. $g''(x) = f'(x)$ changes sign only @ $x = 0$

$\therefore$ g has an inflection pt. @ $x = 0$. 1pt.

d. $\frac{f(3) - f(-4)}{3 + 4} = \underline{\frac{-2}{7}}$ 1pt.

1pt. To use MVT, f must be diff. @ each pt on $(-4, 3)$, but f is not diff. @ $x = -3$, $x = 0$

May 6-8:13 AM

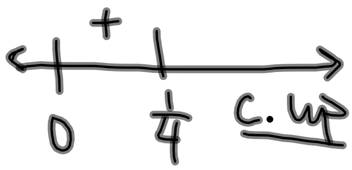
5. a. $(0, 1400)$

$$m = \frac{dW}{dt} = \frac{1}{25}(1400 - 300) = \underline{44} \quad \text{1pt.}$$

$$y = 44(t-0) + 1400 = 44t + 1400$$

$$W\left(\tfrac{1}{4}\right) = 44\left(\tfrac{1}{4}\right) + 1400 = \underline{1411 \text{ tons}} \quad \text{1pt.}$$

b. $$\frac{d^2W}{dt^2} = \frac{1}{25}\left(\frac{dW}{dt}\right)$$
$$= \frac{1}{25}\left(\frac{1}{25}\right)(W - 300) = \underline{\frac{1}{625}(W-300)} \quad \text{1pt.}$$



b/c $W''(t)$ is positive the tangent line is an <u>underestimate.</u> 1pt.

c. $$\frac{dW}{W-300} = \frac{dt}{25} \leftarrow \text{1pt.}$$

$$1 \rightarrow \ln|W - 300| = \frac{1}{25}t + C \quad \text{1pt.}$$

$$1 \rightarrow \ln 1100 = C$$

$$\ln_e|W - 300| = \frac{1}{25}t + \ln 1100$$

$$e^{\frac{1}{25}t + \ln 1100} = W - 300$$

$$\underline{W = 300 + 1100e^{\frac{1}{25}t}} \quad \text{1pt.}$$

6. a. $\lim_{x\to 0^-} f(x) = 1 - 2\sin 0 = 1$

$\lim_{x\to 0^+} f(x) = e^{-4\cdot 0} = 1$

so $\underline{\lim_{x\to 0} f(x) = 1} \longrightarrow f(0) = 1$

2pts. $\therefore$ f is cont. @ $x = 0$

b. $f'(x) = \begin{cases} -2\cos x & , x < 0 \\ -4e^{-4x} & , x > 0 \end{cases}$ 2pts.

$-2\cos x \neq -3$

$-4e^{-4x} = -3$

$e^{-4x} = \frac{3}{4}$

$-4x = \ln \frac{3}{4}$

$\underline{x = \frac{\ln \frac{3}{4}}{-4}}$ 1pt.

c. $\frac{1}{2}\int_{-1}^{1} f(x)\,dx$

$= \frac{1}{2}\left(\underline{\int_{-1}^{0}(1-2\sin x)\,dx + \int_{0}^{1} e^{-4x}\,dx}\right)$
1pt.

$= \frac{1}{2}\left[\underline{(x+2\cos x)}\Big|_{-1}^{0} + \underline{\left(-\frac{1}{4}e^{-4x}\right)}\Big|_{0}^{1}\right]$
1pt. 1pt.

$= \frac{1}{2}\left(3 - 2\cos(-1) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4}\right)\right)$

$= \underline{\frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}}$ 1pt.