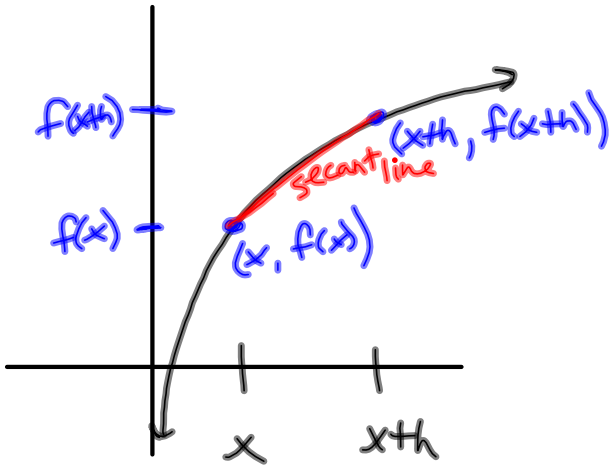
Rev

DQT : $m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

* instantaneous rate of change
slope of the tangent line

f(a) : y-coordinate

Sep 26-11:58 AM



$m: \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$

* shows $h \to 0$

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$f(x) = x^2 \quad x = 3.5$

$$\lim_{h \to 0} \frac{f(3.5+h) - f(3.5)}{h}$$

$$\lim_{h \to 0} \frac{(3.5+h)^2 - 3.5^2}{h}$$

$$\lim_{h \to 0} \frac{12.25 + 7h + h^2 - 12.25}{h}$$

$$\lim_{h \to 0} \frac{h(7+h)}{h} = 7$$

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3.1 The Derivative

RS #24

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Notation

$f'(x)$ "f prime of x"

y' "y prime"

$\frac{dy}{dx}$ The deriv. of y w/ respect to x

$\frac{d}{dx} f(x)$ The deriv. of f at x.

Sep 26-1:04 PM

1. $f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = 2x$$

* it will work for any x!

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2. $f(x) = 2x$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h) - 2x}{h}$$

$$= \lim_{h \to 0} \frac{2x + 2h - 2x}{h}$$

$$= 2$$

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3. $y = \frac{1}{x}$

$$y' = \lim_{h\to 0} \frac{\frac{x}{x}\left(\frac{1}{x+h}\right) - \left(\frac{1}{x}\right)\left(\frac{x+h}{x+h}\right)}{h}$$

$$= \lim_{h\to 0} \frac{x - x - h}{x(x+h)}$$

$$= \lim_{h\to 0} \frac{-h}{x^2+xh} \cdot \frac{h}{1} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-1}{x^2+xh} = -\frac{1}{x^2}$$

or

$$-x^{-2}$$

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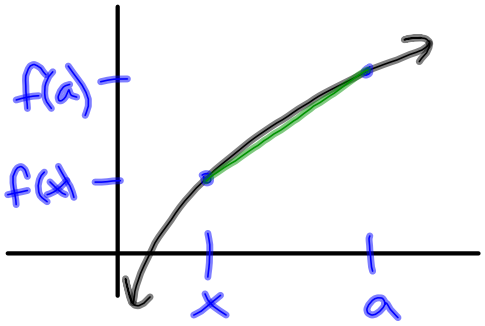
$$f'(x) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$ Slope



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4. $y = x^4$

$y' = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{x^4 - a^4}{x - a}$

$= \lim_{x \to a} \frac{(x^2 - a^2)(x^2 + a^2)}{x - a}$

$= \lim_{x \to a} \frac{(x - a)(x + a)(x^2 + a^2)}{(x - a)}$

$= \lim_{x \to a} (x + a)(x^2 + a^2)$

$= (a + a)(a^2 + a^2)$

$= (2a)(2a^3)$

$= 4a^3$

$y' = 4x^3$

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$y = x^4$

1st Def:

$y' = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$

yuck!

Also: $y = \sqrt{x}$

$y' = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ Can't simplify!

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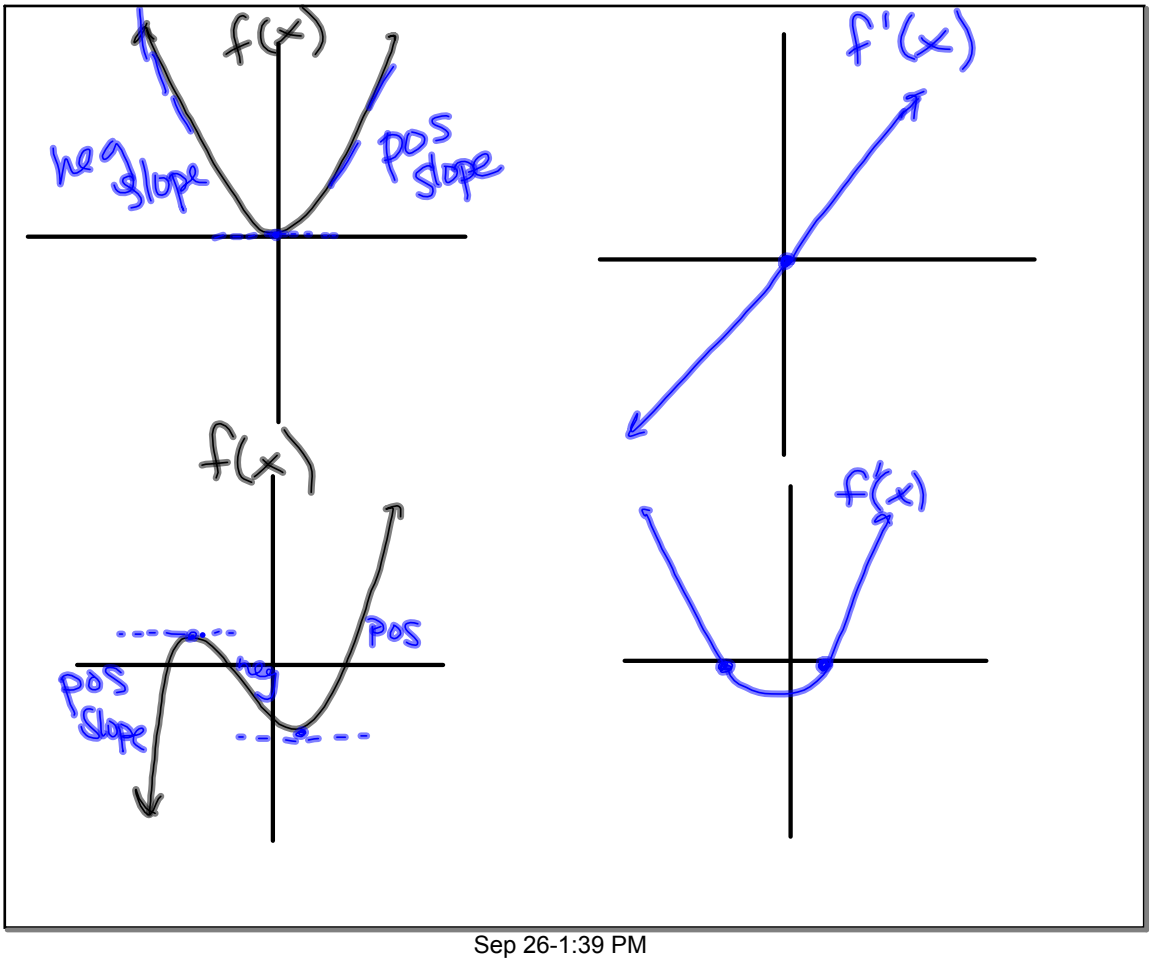

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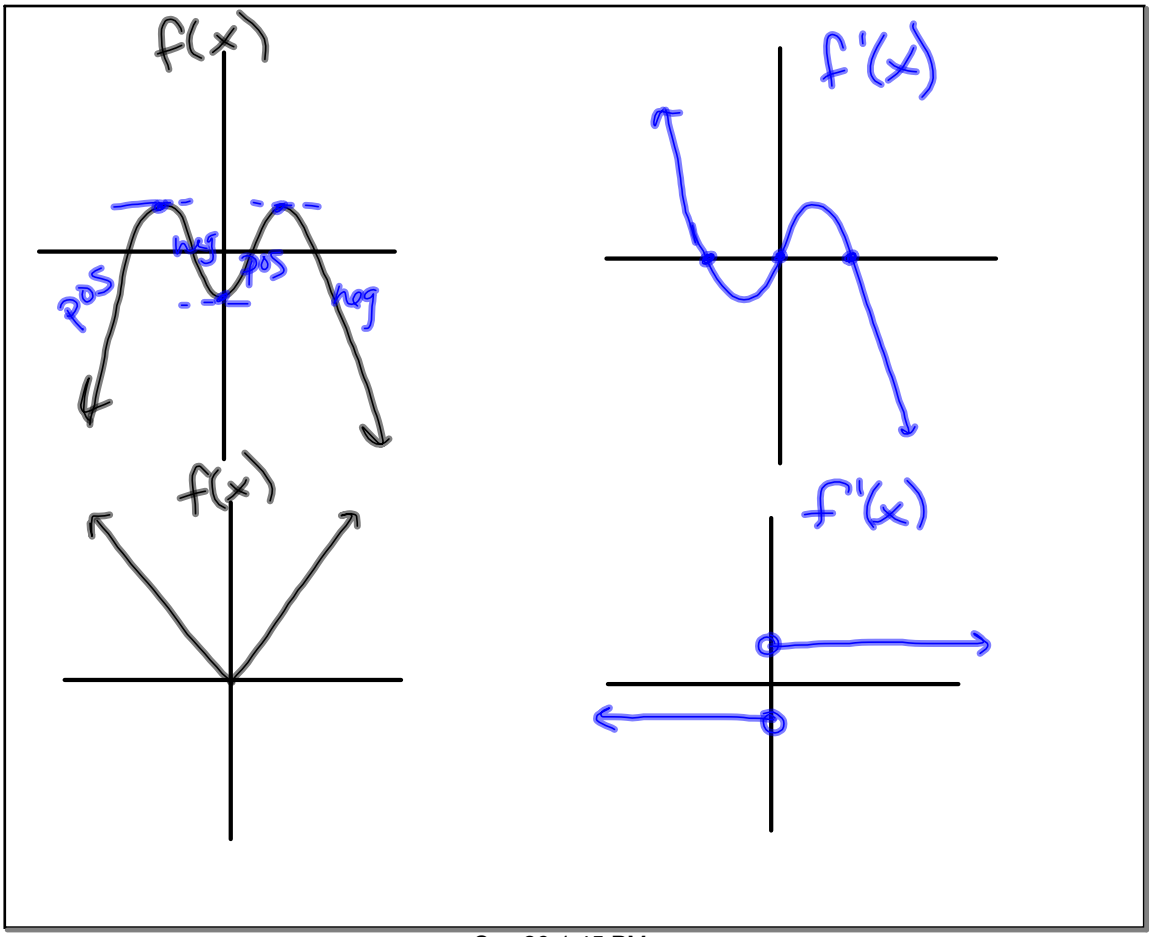

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