3.1 Exponential & Logistic Functions

Obj: 1. Evaluate exponential expressions
2. Identify & graph exponential & logistic fncs.

Exponential fnc: $f(x) = a \cdot b^x$

initial value (a)

base multiplier (b)

3^x
base = 3
initial value: 1
exponential

$6x^{-4}$
not exp.

$-2 \cdot 1.5^x$
exp.
base: 1.5
init. val: -2

Nov 2-9:11 AM

For $f(x)=2^x$, evaluate when $x=4, 0, -3, \frac{1}{2}, -\frac{3}{2}$

$f(4)=2^4=2\cdot2\cdot2\cdot2=16$

$f(0)=2^0=1$

$f(-3)=2^{-3}=\frac{1}{2^3}=\frac{1}{8}$

$f(\frac{1}{2})=2^{\frac{1}{2}}=\sqrt{2}$ $\quad$ $f(\frac{1}{3})=2^{\frac{1}{3}}=\sqrt[3]{2}$

$f(-\frac{3}{2})=2^{-\frac{3}{2}}=\frac{1}{2^{\frac{3}{2}}}=\frac{1}{\sqrt{2^3}}=\frac{1}{\underset{2\cdot4}{\sqrt{8}}}=\frac{1}{2\sqrt{2}}$

"roots in the ground
powers in the sky".

$f(\frac{2}{3})=2^{\frac{2}{3}}=\sqrt[3]{2^2}=\sqrt[3]{4}$

$f(\frac{4}{3})=2^{\frac{4}{3}}=\sqrt[3]{2^4}=\sqrt[3]{\underset{8\cdot2}{16}}=\sqrt[3]{8\cdot2}=2\sqrt[3]{2}$

Nov 2-9:54 AM

x	$g(x)$	$h(x)$
-2	$\frac{4}{9}$	128
-1	$\frac{4}{3}$	32
0	4	8
1	12	2
2	36	$\frac{1}{2}$
3	108	$\frac{1}{8}$

$\cdot 3$ $\cdot 3$ $\cdot 3$ $\cdot 3$ $\cdot 3$

$\cdot \frac{1}{4}$

$g(x) = a \cdot b^x$

$4 \cdot 3^x$

$g(x) = 4 \cdot 3^x$

growth

$h(x) = 8 \cdot \left(\frac{1}{4}\right)^x$

decay

Nov 2-10:03 AM

For any exp. fnc $f(x) = a \cdot b^x$

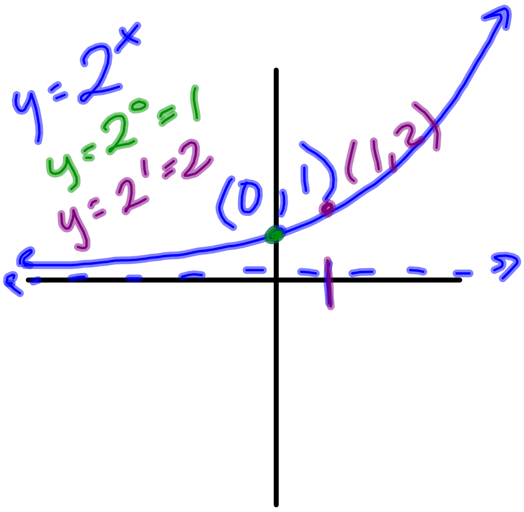
(growth factor)

if $\underline{b} > 1$ fnc is growth

if $\underline{b} < 1$ fnc is decay

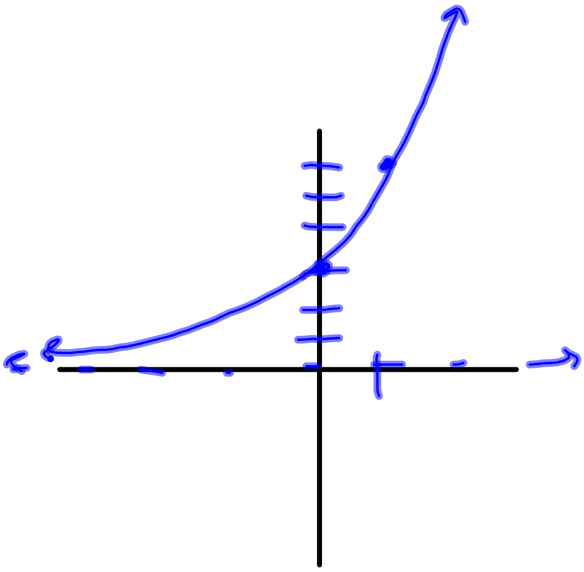
(decay factor)

Nov 2-10:10 AM



$$y = 3 \cdot 2^x$$

x	y
0	3
1	$3 \cdot 2^1 = 6$



$y = 2^x$

$g(x) = 2^{x-1}$

right 1

$h(x) = 2^{-x}$

refl. over y-axis

$k(x) = 4 \cdot 2^x$

v. str. by 4

"Natural Base"

$f(x) = e^x$

$g(x) = e^{2x}$
h. comp. by $\frac{1}{2}$

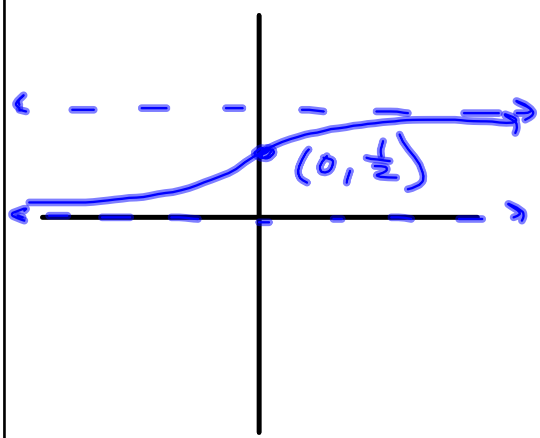
$h(x) = -e^x$
refl. over x axis

$k(x) = \frac{1}{3}e^x$
V. comp. by $\frac{1}{3}$

Logistic Fnc

limit to growth

$$f(x) = \frac{c}{1 + a \cdot b^{x}} \quad \text{or} \quad f(x) = \frac{c}{1 + a \cdot e^{-kx}}$$

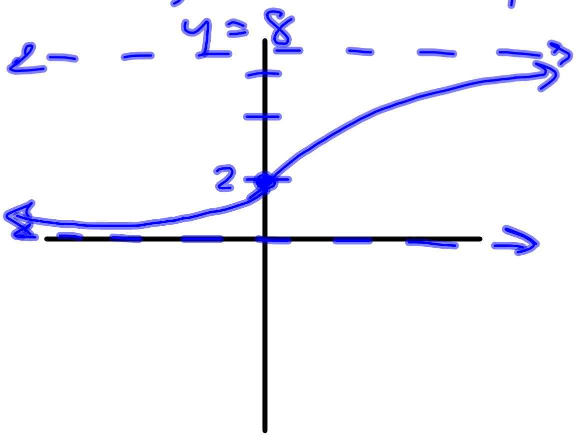


$$f(x) = \frac{8}{1+3\cdot 7^x}$$

Find y intercept & the horizontal asymptotes

HA: $y = 0$, $y = 8$

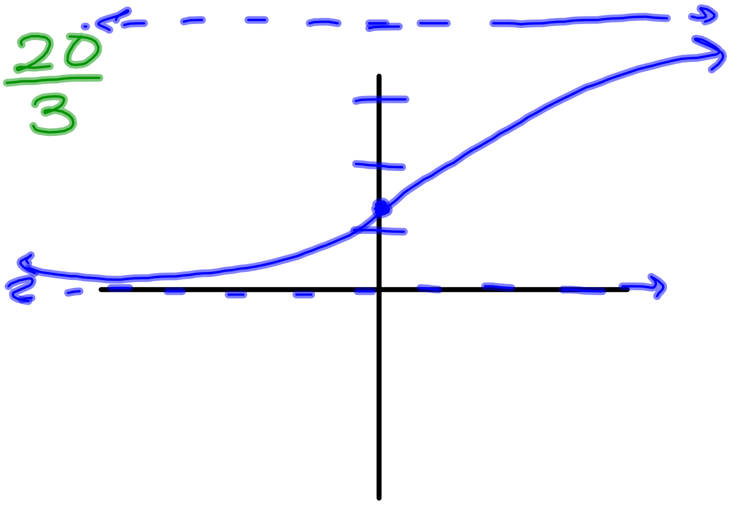
y int: $y = \frac{8}{1+3\cdot 7^0} = \frac{8}{1+3\cdot 1} = \frac{8}{4} = 2$



$$f(x) = \frac{20}{1+2e^{-3x}}$$

HA: $y = 0$ $y = 20$

yint: $y = \frac{20}{1+2e^{0}} = \frac{20}{3}$



Nov 2-10:23 AM

Population of San Jose:

1990	782,248
2000	895,193

$10\{$

If the pop. of San Jose is exponential, when will the pop. be 1 million?

$$P(t) = P_0 \cdot b^t$$

$$\frac{895{,}193}{782248} = \frac{\cancel{782{,}248} \cdot b^{10}}{\cancel{782248}}$$

$$\sqrt[10]{b^{10}} = \sqrt[10]{\frac{895193}{782248}} = \left(\frac{895193}{782248}\right)^{\left(\frac{1}{10}\right)}$$

$$b \approx 1.0136$$

$$P(t) = 782248(1.0136)^t$$

$$\frac{1000000}{782248} = \frac{\cancel{782248}(1.0136)^t}{\cancel{782248}}$$

$$\ln 1.0136^t = \ln \frac{1000000}{782248}$$

$$\frac{t \ln 1.0136}{\cancel{\ln 1.0136}} = \frac{\ln\left(\frac{1000000}{782248}\right)}{\ln 1.0136}$$

$$t \approx 18.2 \text{ yrs.}$$

in 2008