3.2 Differentiability

$f(x) = \frac{1}{2}x^2 \quad [1,5]$

$$\frac{f(5)-f(1)}{5-1} = \frac{\frac{25}{2} - \frac{1}{2}}{4} = 3$$

$[2,4] : \frac{f(4)-f(2)}{4-2} = 3$

$[2.5, 3.5] \quad = 3$

$[3-h, 3+h]$

$$[x-h, x+h] = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3-h)}{2h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2}(3+h)^2 - \frac{1}{2}(3-h)^2}{2h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2}(9+6h+h^2) - \frac{1}{2}(9-6h+h^2)}{2h}$$

$$= \lim_{h \to 0} \frac{\cancel{\frac{9}{2}} + 3h + \cancel{\frac{1}{2}h^2} - \cancel{\frac{9}{2}} + 3h - \cancel{\frac{1}{2}h^2}}{2h}$$

$$= \lim_{h \to 0} \frac{6\cancel{h}}{2\cancel{h}} = 3$$

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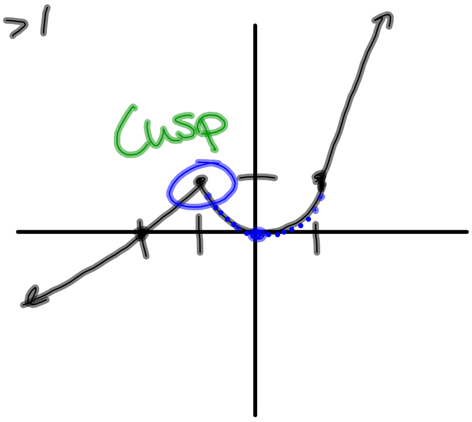
$$f(x) = \begin{cases} x+2 & x<-1 \\ x^2 & -1 \le x \le 1 \\ 2x-1 & x>1 \end{cases}$$



$x=-1$ point of change $\quad$ $x=0$ curve

Definition:

differentiable if $f'(x)$ exists.

LHD = RHD

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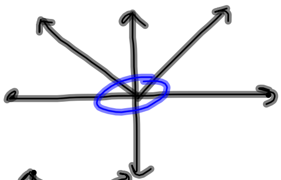
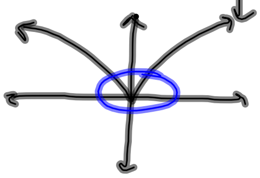
Not Differentiable:

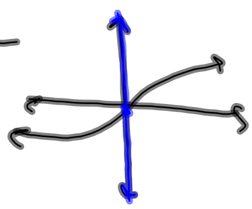
1. Corner: $y=|x|$



2. Cusp: $y=x^{\frac{2}{3}}$



3. Discontinuity: hole, asymptote
4. Vertical Tangent: $y=\sqrt[3]{x}$



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Compare

$f(x) = \sqrt{x^2 + .001} + .99$

$g(x) = |x| + 1$

Zoom in: (0,1) center

3 xs

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If a fnc is differentiable, then it is continuous.

Differentiablity $\Rightarrow$ Continuity

Does continuity $\Rightarrow$ differentiability?

NO!

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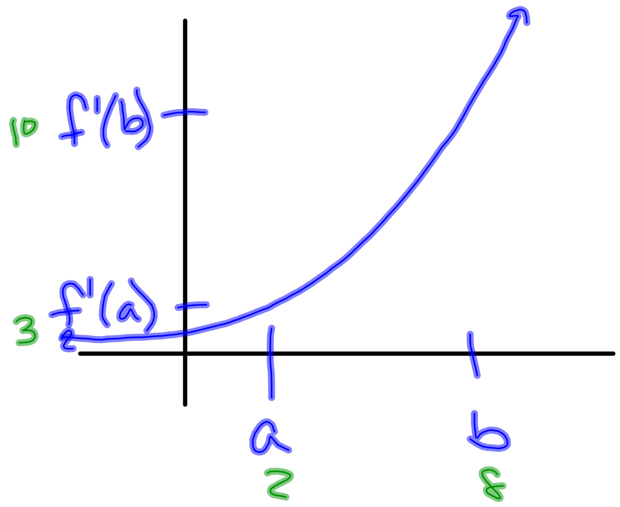
IVT for Derivatives



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