3.2 Exponential & Logistic Modeling

Obj: 1. Be able to use exp. growth, decay, and regression to model real-life problems.

on calc.

word problems

$P(t) = P_0 \cdot b^t$

.025

A pop. is inc. by 2.5% yearly.

b = ? 1.025

b = 1 + r

100%

Determine if the fnc. is growth or decay.
Find the rate.

a. $P(t) = 782,248 \cdot \underline{1.0136}^t$

growth

$b = 1 + \underline{r} = 1.0136$

$\quad -1 \qquad -1$

$r = .0136$

1.36%

Nov 6-9:50 AM

b. $P(t) = 1{,}203{,}368 \cdot \underline{0.9858}^{t}$

decay

$$\begin{aligned} 1 + r &= .9858 \\ -1 \quad & \quad -1 \\ r &= -.0142 \end{aligned}$$

decay

-1.42% or 1.42% decay

c. $P(t) = 4.3 \cdot 1.018^t$

growth

$$1 + r = 1.018$$
$$-1 \qquad\qquad -1$$
$$r = .018$$

1.8% growth

Determine the exponential fnc. with initial value 12 increasing at a rate of 8% per year.

.08

$P(t) = P_0 \cdot (1+r)^t$

$P(t) = 12(1.08)^t$

Determine the exp. fnc. w/ init val. 5 decreasing at a rate of .59% per week.

$P(t) = 5 \cdot .9941^t$

$(1-.0059)$

Suppose a culture of 100 bacteria is put into a petri dish and doubles every hour. Predict when (t) # of bacteria will be 350,000.

$P(t) = P_0 \cdot (1+r)^t$

* $P(t) = 100(2)^t$

$\frac{3500\cancel{00}}{1\cancel{00}} = \frac{100(2)^t}{100}$

$\ln 3500 = \ln 2^t$

$\frac{\ln 3500}{\ln 2} = \frac{t \cancel{\ln 2}}{\cancel{\ln 2}}$

$t = \frac{\ln 3500}{\ln 2}$

≈ 11.77 hours

Half-Life

$$P(t) = P_0\left(\frac{1}{2}\right)^{t/k}$$

k: length of half life

The half life of a radioactive substance is 20 days. If there are 5 g present init. how long until 1 g is left?

$$P(t) = 5\left(\frac{1}{2}\right)^{\frac{t}{20}}$$

$$\frac{1}{5} = \frac{\cancel{5}\left(\frac{1}{2}\right)^{\frac{t}{20}}}{\cancel{5}}$$

$$\ln\frac{1}{5} = \frac{t}{20}\ln\left(\frac{1}{2}\right)$$

$$20\,\frac{\ln .2}{\ln .5} = \frac{\cancel{20}\,t}{\cancel{20}}\,\frac{\cancel{\ln .5}}{\cancel{\ln .5}}$$

$$t = 20\left(\frac{\ln .2}{\ln .5}\right)$$

$$\approx 46.44 \text{ days}$$

P 293 Ex 6

$y = a * b^x$

$a = 80.55$

$b = 1.01289$

$\underline{y = 80.55(1.01289)^x}$

Predict pop in 2010.

$y = 80.55(1.01289)^{110}$

$= 329.55$ million

1200 students at a school.

Bob, Carol, Ted, Alice start a rumor that spreads logistically so that

$S(t) = \frac{1200}{1+39e^{-.9t}}$ models the # of

students who have heard it.

How many have heard it by the end of Day 0?

$t=0$

$S(0) = \frac{1200}{1+39\cancel{e^{-.9\cdot 0}}_{1}} = \frac{120\cancel{0}}{4\cancel{0}} = 30$

How long will it take for 1000 people to hear?

$\overset{1000}{S(t)} = \frac{1200}{1+39e^{-.9t}}$

$y=1000$

$y = \frac{1200}{1+39e^{-.9x}}$ → intersection: 5.86 days