3.5

1. $y = \frac{3}{x^2} = 3x^{-2}$

$y' = -6x^{-3}$ or $\frac{-6}{x^3}$

2. $f(x) = 3e^5$

$f'(x) = 0$

3. $g(x) = \frac{x^6}{3} = \frac{1}{3}x^6$

$g'(x) = 6\left(\frac{1}{3}x^5\right) = 2x^5$

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4. $y = \frac{x^3}{x+1}$

$y' = \frac{(x+1)(3x^2) - x^3(1)}{(x+1)^2} = \frac{3x^3+3x^2-x^3}{(x+1)^2} = \frac{2x^3+3x^2}{(x+1)^2}$

5. $f(x) = x^3 - 3x \quad x = 4$

$f'(x) = 3x^2 - 3$

$f'(4) = 3(4)^2 - 3 = 45$

x	y
4	$4^3 - 12 = 52$

$y - 52 = 45(x-4)$

6. $f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3 = 0$

$+3 \quad +3$

$\frac{3}{3} \quad \frac{+3}{3}$

$x^2 = 1$

$x = \pm 1$

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Numerical Derivative

$y = x^3 - 4x - 1$

Find the deriv. @ $x = 2$.

$y' = 3x^2 - 4 \Big|_{x=2} = 3(2)^2 - 4 = 8$

On calc: Math → opt. 8

nderiv(fnc, variable, # value)

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$f(x) = \sin x$

Graphically: $y = \text{nderiv}(\sin x, x, x)$

$f'(x) = \cos x$

Analytically: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

$\lim_{h \to 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$

$\lim_{h \to 0} \frac{\sin x(\cosh - 1) + \cos x \sinh}{h}$

$\lim_{h \to 0} \sin x \left(\frac{\cosh - 1}{h}\right) + \cos x \left(\frac{\sinh}{h}\right)$

$\sin x \cdot 0 + \cos x \cdot 1$

$= \cos x$

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$f(x) = \cos x$

Graphically: $y = nderiv(\cos x, x, x)$

$f'(x) = -\sin x$

$f(x) = \tan x = \frac{\sin x}{\cos x}$

analytically: $f'(x) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

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RS #61-66

61. $\frac{d}{dx}(\sin x) = \cos x$

62. $\frac{d}{dx}(\cos x) = -\sin x$

63. $\frac{d}{dx}(\tan x) = \sec^2 x$

64. $\frac{d}{dx}(\cot x) = -\csc^2 x$

65. $\frac{d}{dx}(\sec x) = \sec x \tan x$

66. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

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1. $y = x^3 \cdot \cos x$

$y' = x^3(-\sin x) + \cos x(3x^2)$

$= -x^3 \sin x + 3x^2 \cos x$

2. $f(x) = \dfrac{\csc x}{x^3}$

$f'(x) = \dfrac{x^3(-\csc x \cot x) - \csc x(3x^2)}{x^6}$

$= \dfrac{-x^3 \csc x \cot x - 3x^2 \csc x}{x^6}$

$\dfrac{x^2(-x\csc x \cot x - 3\csc x)}{x^2 \cdot x^4} = \dfrac{-x\csc x \cot x - 3\csc x}{x^4}$

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3. $y = \dfrac{\tan x}{x} \qquad x = \dfrac{\pi}{4}$

$y' = \dfrac{x\sec^2 x - \tan x}{x^2}\Big|_{x=\frac{\pi}{4}} = \dfrac{\frac{\pi}{4}\sec^2\frac{\pi}{4} - \tan\frac{\pi}{4}}{\left(\frac{\pi}{4}\right)^2}$

$= \dfrac{\frac{\pi}{4}(\sqrt{2})^2 - 1}{\frac{\pi^2}{16}} = \dfrac{\frac{\pi}{2} - 1}{\frac{\pi^2}{16}}$

x	y
$\frac{\pi}{4}$	$\frac{\tan\frac{\pi}{4}}{\frac{\pi}{4}}$

$\dfrac{1}{\frac{\pi}{4}} = \dfrac{4}{\pi}$

$y - \dfrac{4}{\pi} = \dfrac{\frac{\pi}{2} - 1}{\frac{\pi^2}{16}}\left(x - \dfrac{\pi}{4}\right)$

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5. $s = 1 - 4\cos t$

a. $v(t) = -4(-\sin t) = 4\sin t$

speed $= |4\sin t|$

$a(t) = 4\cos t$

b. $s\left(\frac{\pi}{4}\right) = 1 - 4\cos\frac{\pi}{4} = 1 - 4\left(\frac{\sqrt{2}}{2}\right) = 1 - 2\sqrt{2}$

$v\left(\frac{\pi}{4}\right) = 4\sin\frac{\pi}{4} = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

Speed $= |2\sqrt{2}| = 2\sqrt{2}$

$a\left(\frac{\pi}{4}\right) = 4\cos\frac{\pi}{4} = 2\sqrt{2}$

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