3.6 Day 2

1. $f(x) = \sin^2 x = (\sin x)^2$ $f(x) = (x)^2$
$g(x) = \sin x$

$f'(x) = 2(\sin x) \cdot \cos x = 2 \sin x \cos x$

2. $g(x) = \dfrac{3}{(x^2+1)^2}$ $f(x) = \dfrac{3}{(x)^2} = 3(x)^{-2}$
$g(x) = x^2+1$

$g'(x) = -6(x^2+1)^{-3} \cdot 2x = \dfrac{-12x}{(x^2+1)^3}$

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$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

If $y = f(u)$, then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$\dfrac{d}{dx}(\sin x) = \cos x$

$\dfrac{d}{dx}(\sin u) = \cos u \cdot \dfrac{du}{dx}$

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1. $y=(\csc x+\cot x)^{-1}$

$f(x)=(x)^{-1}$
$g(x)=\csc x+\cot x$

$y'=-1(\csc x+\cot x)^{-2}\cdot(-\csc x\cot x-\csc^2 x)$

$=\dfrac{-1(-\csc x\cot x-\csc^2 x)}{(\csc x+\cot x)^2}=\dfrac{\csc x\cot x+\csc^2 x}{(\csc x+\cot x)^2}$

$\dfrac{\csc x\,\cancel{(\cot x+\csc x)}}{(\csc x+\cot x)^{\cancel{2}}}=\dfrac{\csc x}{\csc x+\cot x}$

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2. $f(x)=x^3\cdot\underline{(2x-5)^4}$ chain

$f(x)=(x)^4$
$g(x)=2x-5$

$f'(x)=x^3(4(2x-5)^3\cdot 2)+(2x-5)^4\cdot 3x^2$
$=8x^3(2x-5)^3+3x^2(2x-5)^4$

3. $y=4\sqrt{\sec x+\tan x}$

$f(x)=\sqrt{(x)}$
$g(x)=\sec x+\tan x$

$y'=\cancel{{}^2 4}\left(\dfrac{1}{\cancel{2}\sqrt{\sec x+\tan x}}\right)(\sec x\tan x+\sec^2 x)$

$=\dfrac{2(\sec x\tan x+\sec^2 x)}{\sqrt{\sec x+\tan x}}$

$\dfrac{2\sec x(\tan x+\sec x)^1}{(\sec x+\tan x)^{\frac{1}{2}}}$

$2\sec x(\tan x+\sec x)^{1-\frac{1}{2}}$

$\boxed{2\sec x\sqrt{\tan x+\sec x}}$

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4. $g(x)=\dfrac{x}{\left(\sqrt{1+x^2}\right)}$ chain

$f(x)=\sqrt{(x)}$
$g(x)=1+x^2$

$$g'(x)=\frac{\sqrt{1+x^2}\cdot 1-x\left(\frac{1}{\cancel{2}\sqrt{1+x^2}}\right)(\cancel{2}x)}{\left(\cancel{\sqrt{1+x^2}}\right)^{\cancel{2}}}$$

$$=\frac{\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{1+x^2}$$

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5. $y=(1+\cos(2x))^2$

$f(x)=(x)^2$
$g(x)=1+\cos(x)$
$h(x)=2x$

$y'=2(1+\cos 2x)\cdot(-\sin 2x)\cdot(2)$
$=-4\sin 2x(1+\cos 2x)$

6. $y=\sqrt{\tan(5x)}$

$f(x)=\sqrt{(x)}$
$g(x)=\tan(x)$
$h(x)=5x$

$$y'=\frac{1}{2\sqrt{\tan 5x}}\cdot \sec^2 5x\cdot 5$$

$$=\frac{5\sec^2 5x}{2\sqrt{\tan 5x}}$$

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7. $r = \underbrace{\sec(2\theta)}\underbrace{\tan(2\theta)}$

$$\frac{dr}{d\theta} = \sec 2\theta \cdot \sec^2 2\theta \cdot 2 + \tan 2\theta \cdot \sec 2\theta \tan 2\theta \cdot 2$$

$$= 2\sec^3 2\theta + 2\tan^2 2\theta \sec 2\theta$$

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8. $f(x) = \cot x$

$f'(x) = -\csc^2 x = -(\csc x)^2$ $\quad f(x) = -(x)^2$, $g(x) = \csc x$

$f''(x) = -2(\csc x)(-\csc x \cot x)$

$= 2\csc^2 x \cot x$

9. $f(x) = 9\tan\left(\frac{x}{3}\right)$ $\quad f(x) = 9\tan x$, $g(x) = \frac{1}{3}x$

$f'(x) = 9\sec^2\left(\frac{1}{3}x\right) \cdot \frac{1}{3}$

$= 3\sec^2\left(\frac{1}{3}x\right) = 3\left(\sec\left(\frac{1}{3}x\right)\right)^2$ $\quad f(x) = 3(x)^2$, $g(x) = \sec(x)$, $h(x) = \frac{1}{3}x$

$f''(x) = 6\left(\sec\frac{1}{3}x\right) \cdot \left(\sec\frac{1}{3}x \tan\frac{1}{3}x\right) \cdot \frac{1}{3}$

$= 2\sec^2\left(\frac{1}{3}x\right)\tan\left(\frac{1}{3}x\right)$

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58. b. $f(x) \cdot g^3(x)$, $x=0$

$f(x) \cdot (g(x))^3$ product & chain

Deriv: $f(x) \cdot [3(g(x))^2 \cdot g'(x)] + (g(x))^3 \cdot f'(x)$

$1\left(3(1)^2 \cdot \frac{1}{3}\right) + 1^3 \cdot 5 = 6$

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