3.7

$x^2+y^2=9$

1. Find the equation of the tangent line through $(2,-\sqrt{5})$.

$\frac{d}{dx}(x^2+y^2=9)$

$2x+2y\frac{dy}{dx}=0$

$-2x \qquad -2x$

$\frac{2y\frac{dy}{dx}}{2y}=\frac{-2x}{2y}$

$\frac{dy}{dx}=-\frac{x}{y}\Big|_{(2,-\sqrt{5})}=\frac{-2}{+\sqrt{5}}$

$y+\sqrt{5}=\frac{2}{\sqrt{5}}(x-2)$

Oct 24-12:46 PM

Steps

1. Take the deriv. w/ respect to x.
2. Solve for $\frac{dy}{dx}$.

2. $y^2=x \qquad (4,-2)$

$2y\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{1}{2y}\Big|_{(4,-2)}=\frac{1}{-4}$

tangent: $y+2=-\frac{1}{4}(x-4)$

normal: $y+2=4(x-4)$

Oct 24-1:45 PM

3. $x^2 - y^2 = 25 \quad (-5, 0)$

$2x - 2y\frac{dy}{dx} = 0$

$\frac{-2y\frac{dy}{dx}}{-2y} = \frac{-2x}{-2y}$

$\frac{dy}{dx} = \frac{x}{y}\Big|_{(-5,0)} = \frac{-5}{0}$ undefined

Vertical Line! $x = -5$

normal: $y = 0$

Oct 24-1:50 PM

4. $x^2 = \frac{x^2+1}{y^2}$

$2x = \frac{y^2(2x) - (x^2+1)(2y\frac{dy}{dx})}{y^4}$

$y^3(2x) = \left(\frac{y(2xy - (2x^2+2)\frac{dy}{dx})}{y^{\cancel{4}3}}\right) y^3$

$2xy^3 = 2xy - (2x^2+2)\frac{dy}{dx}$

$-2xy \quad -2xy$

$\frac{2xy^3 - 2xy}{-(2x^2+2)} = \frac{-(2x^2+2)\frac{dy}{dx}}{-(2x^2+2)}$

$\frac{dy}{dx} = \frac{2(xy^3 - xy)}{2(-x^2-1)} = \frac{xy^3 - xy}{-x^2-1}$

Oct 24-1:53 PM

5. $y^2 = \frac{x^2-1}{x^3}$

$2y\frac{dy}{dx} = \frac{x^3(2x)-(x^2-1)(3x^2)}{x^6}$

$2y\frac{dy}{dx} = \frac{2x^4-3x^4+3x^2}{x^6} = \frac{-x^4+3x^2}{x^6}$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{x^2(-x^2+3)}{x^2\cdot x^4}\cdot\frac{1}{2y}$

$\frac{dy}{dx} = \frac{-x^2+3}{2x^4y}$

Oct 24-1:53 PM

6. $x^2+2x\cdot y+y^2=0$

Product

$2x+(2x\cdot\frac{dy}{dx}+y\cdot 2)+2y\frac{dy}{dx}=0$

$2x\frac{dy}{dx}+2y\frac{dy}{dx} = -2x-2y$

$\frac{\frac{dy}{dx}(2x+2y)}{2x+2y} = \frac{-2x-2y}{2x+2y} = \frac{-2(x+y)}{2(x+y)} = -1$

Oct 24-2:06 PM

7. Find $\frac{d^2y}{dx^2}$

$y^2 = x^2 + 2x$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{2x+2}{2y}$

$\frac{dy}{dx} = \frac{x+1}{y}$

$\frac{d^2y}{dx^2} = \frac{y(1) - (x+1)\frac{dy}{dx}}{y^2}$

$= \frac{y - (x+1)\left(\frac{x+1}{y}\right)}{y^2}$

Oct 24-2:10 PM