

3.8

Part I.

a.  $f(x) = x^2, x \geq 0$

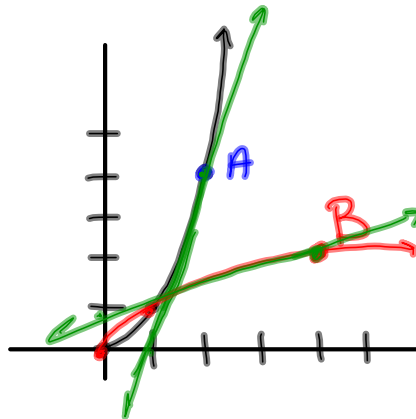
b.  $f(2) = 4 \quad (2, 4)$

d.  $f'(x) = 2x$   
 $f'(2) = 2 \cdot 2 = 4$

f.  $x = y^2$   
 $y = \sqrt{x} = f^{-1}(x)$

i.  $(f^{-1})'(x) = \frac{1}{2\sqrt{x}} \Big|_{x=4} = \frac{1}{4}$

k. reciprocals



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$$(a, b) \longrightarrow (b, a)$$

$$f(a) = b \longrightarrow f^{-1}(b) = a$$

$$f'(a) = c \longrightarrow (f^{-1})'(b) = \frac{1}{c}$$

1.  $f(x) = x^4 - 3x^2 + 4x + 2$

a.  $f(1) = 1 - 3 + 4 + 2 = 4 \quad (1, 4)$

$$f'(x) = 4x^3 - 6x + 4$$

$$f'(1) = 4 - 6 + 4 = 2$$

b.  $f^{-1}(4) = 1 \quad (4, 1)$

$$(f^{-1})'(4) = \frac{1}{2}$$

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$$2. f(x) = \cos x + x$$

$$a. f(0) = \cos 0 + 0 = 1 \quad (0, 1)$$

$$f'(x) = -\sin x + 1$$

$$f'(0) = -\sin 0 + 1 = 1$$

$$b. f^{-1}(1) = 0 \quad (1, 0)$$

$$(f^{-1})'(1) = 1$$

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Part II.

$$\sin y = \sin^{-1} x$$

$$\sin y = x$$

$$\frac{\cos y \frac{dy}{dx}}{\cos y} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \sec y$$

$$\tan y = \tan^{-1} x$$

$$\tan y = x$$

$$\frac{\sec^2 y \frac{dy}{dx}}{\sec^2 y} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \cos^2 y$$

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$$67. \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$68. \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$

$$69. \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$70. \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$$

$$71. \frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$72. \frac{d}{dx}(\csc^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

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$$3. f(x) = \cos^{-1}(3x)$$

$$f'(x) = \frac{-1}{\sqrt{1-(3x)^2}} \cdot 3 = \frac{-3}{\sqrt{1-9x^2}}$$

$$4. y = \cos^{-1}\left(\frac{3}{x}\right) = \sec^{-1}\left(\frac{x}{3}\right)$$

$$y' = \frac{1}{\left|\frac{x}{3}\right|\sqrt{\left(\frac{x}{3}\right)^2-1}} \cdot \frac{1}{3} = \frac{1}{\cancel{3}\left|\frac{x}{3}\right|\sqrt{\frac{x^2}{9}-1}}$$

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$$5. g(x) = \sin^{-1} x^2$$

$$g'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

$$6. g(x) = \cos^{-1} \frac{4}{x^2} = \sec^{-1} \left( \frac{x^2}{4} \right)$$

$$g'(x) = \frac{1}{\left| \frac{x^2}{4} \right| \sqrt{\left( \frac{x^2}{4} \right)^2 - 1}} \cdot \frac{1}{2} x$$

$$= \frac{x}{2 \left| \frac{x^2}{4} \right| \sqrt{\frac{x^4}{16} - 1}}$$

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$$7. f(x) = \csc^{-1}(3x+2)$$

$$f'(x) = \frac{-1}{|3x+2| \sqrt{(3x+2)^2 - 1}} \cdot 3 = \frac{-3}{|3x+2| \sqrt{9x^2 + 12x + 3}}$$

$$8. y = \tan^{-1} \sqrt{x^2+2}$$

$$y' = \frac{1}{1 + (\sqrt{x^2+2})^2} \cdot \frac{1}{2\sqrt{x^2+2}} \cdot 2x$$

$$= \frac{x}{(x^2+3)\sqrt{x^2+2}}$$

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$$9. y = \sec^{-1}(3x^2)$$

$$y' = \frac{1}{|3x^2|\sqrt{(3x^2)^2-1}} \cdot 6x = \frac{\cancel{2}6x}{|3x^2|\sqrt{9x^4-1}}$$
$$\frac{2}{x\sqrt{9x^4-1}}$$

$$10. f(x) = \sin^{-1}\left(\frac{1}{x}\right) = \csc^{-1}x$$

$$f'(x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

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