3.9

$$y = e^x$$

$$f(x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h} = \lim_{h \to 0} \frac{e^x \cdot (e^h - 1)}{h}$$

$$= \lim_{h \to 0} e^x \cdot \lim_{h \to 0} \frac{e^h - 1}{h}$$ Graph!

$$= e^x \cdot 1 = e^x$$

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Graphically

$y =$ nDeriv(fnc, x, x)

looks like: e^x

Numerically

Table

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RS

58. $\frac{d}{dx}(e^x) = e^x$

59. $\frac{d}{dx}(a^x) = a^x \ln a$

$y = \ln x$

$e^y = x$

$e^y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$

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RS

56. $\frac{d}{dx}(\ln x) = \frac{1}{x}$, $x > 0$

57. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$, $x > 0$

1. $f(x) = \underline{2} \cdot e^{(2x)}$ Chain Rule

$f'(x) = 2(e^{2x} \cdot 2)$

$= 4e^{2x}$

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2. $y = e^{\left(-\frac{x}{2}\right)}$

$\frac{dy}{dx} = e^{-\frac{x}{2}} \cdot -\frac{1}{2} = -\frac{1}{2}e^{-\frac{x}{2}}$

3. $g(x) = 3^{(4x)}$

$g'(x) = 3^{4x}\ln 3 \cdot 4 = 3^{4x} \cdot 4\ln 3 = 3^{4x}\ln 81$

$= (3^4)^x \ln 81$

$= 81^x \ln 81$

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4. $y = x \cdot e^{(2x)}$

$\frac{dy}{dx} = x \cdot \underbrace{(e^{2x} \cdot 2)}_{\text{chain Rule}} + e^{2x} \cdot 1$

$= 2xe^{2x} + e^{2x}$

5. $y = 4^{(x^2)}$

$\frac{dy}{dx} = 4^{x^2}\ln 4 \cdot 2x$

$= x\, 4^{x^2}\ln 16$

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6. $g(x) = \ln\left(\frac{1}{x}\right) = \ln(x^{-1}) = -\ln x$

$g'(x) = \frac{-1}{x}$

OR: $\frac{1}{\frac{1}{x}} \cdot -x^{-2} = \frac{-1}{\cancel{\frac{1}{x}} \cdot x^{2}} = \frac{-1}{x}$

7. $y = \ln(x^2) = 2\ln x$

$\frac{dy}{dx} = \frac{2}{x}$

OR: $\frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x} = \frac{2}{x}$

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8. $y = \log_6(x^3 + 2x - 1)$

$\frac{dy}{dx} = \frac{1}{(x^3+2x-1)\ln 6} \cdot (3x^2+2)$

$= \frac{3x^2+2}{\ln 6(x^3+2x-1)}$

9. $y = \frac{e^x}{\ln x}$

$\frac{dy}{dx} = \frac{\ln x \cdot e^x - e^x \cdot \frac{1}{x}}{\ln^2 x} = \frac{\frac{x}{x}(e^x \ln x) - \frac{e^x}{x}}{\ln^2 x}$

$= \frac{\frac{xe^x \ln x - e^x}{x}}{\cancel{\ln^2 x}} \cdot \frac{1}{\ln^2 x}$

$= \frac{xe^x \ln x - e^x}{x \ln^2 x}$

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10. $y = x\ln x$

$\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1 = 1 + \ln x$

11. $y = e^{2\ln 6}$ constant!

$\frac{dy}{dx} = 0$

12. $f(x) = \ln 2^{x} = x\ln 2$

$f'(x) = \ln 2$

OR : $\frac{1}{\cancel{2^x}} \cdot \cancel{2^x}\ln 2 = \ln 2$

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13. $y = \log_6 \sqrt[3]{x} = \log_6 x^{\frac{1}{3}} = \frac{1}{3}\log_6 x$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{1}{x\ln 6}\right) = \frac{1}{3x\ln 6}$

OR : $\frac{1}{x^{\frac{1}{3}}\ln 6} \cdot \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{1}{3}}x^{\frac{2}{3}}\ln 6} = \frac{1}{3x\ln 6}$

14. $y = \ln(\cos x)$

$\frac{dy}{dx} = \frac{1}{\cos x} \cdot (-\sin x) = -\tan x$

15. $y = \ln(\ln x)$

$\frac{dy}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x\ln x}$

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16. $y = e^{\tan 2x}$

$\frac{dy}{dx} = e^{\tan 2x} \cdot \sec^2 2x \cdot 2$

$= 2e^{\tan 2x} \sec^2 2x$

17. $f(x) = \sec^{-1}(e^x)$

$f'(x) = \frac{1}{|e^x|\sqrt{(e^x)^2-1}} \cdot e^x = \frac{\cancel{e^x}}{|\cancel{e^x}|\sqrt{e^{2x}-1}}$

$= \frac{1}{\sqrt{e^{2x}-1}}$

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18. $y = 2e^x$ @ $x = 1$

$m: \frac{dy}{dx} = 2e^x \Big|_{x=1} = 2e$

x	y
1	2e

$y = 2e(x-1) + 2e$

19. $y = x^2 \cdot \ln x$ @ $x = e$

$m: \frac{dy}{dx} = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$

$= x + 2x\ln x \Big|_{x=e} = e + 2e\cancel{\ln e} = 3e$

x	y
e	e^2

$y = 3e(x-e) + e^2$

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