

4.3 Day 1

$$f(x) = x^4 - 2x^3 + 2$$

$$f'(x) = 4x^3 - 6x^2 = 0$$

$$2x^2(2x-3) = 0$$

$$2x^2 = 0 \quad 2x-3 = 0$$

$$x = 0 \quad x = \frac{3}{2} \text{ min}$$

inc: $(\frac{3}{2}, \infty)$

dec: $(-\infty, 0) \cup (0, \frac{3}{2})$

	$(-\infty, 0)$	0	$(0, \frac{3}{2})$	$\frac{3}{2}$	$(\frac{3}{2}, \infty)$
Sign of f'	neg	0	neg	0	pos
f: inc/dec	dec	CP	dec	CP	inc

no sign change

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$f(x)$: value of the func.

$f'(x)$: slope \rightarrow where $f(x)$ is inc/dec.

$f''(x)$: concavity \rightarrow up \rightarrow down

Concave Up:

$$f''(x) > 0$$

$$f'(x) \text{ is inc.}$$

Inflection Point
where concavity changes
 $f''(x) = 0$

Concave Down:

$$f''(x) < 0$$

$$f'(x) \text{ is dec.}$$

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Concavity Test

$$f(x) = x^4 - 2x^3 + 2$$

$$f'(x) = 4x^3 - 6x^2$$

$$f''(x) = 12x^2 - 12x = 0$$

$$12x(x-1) = 0$$

$$12x = 0 \quad x - 1 = 0$$

$$x = 0 \quad x = 1 \quad \text{IP}$$

C. up $(-\infty, 0) \cup (1, \infty)$
 C. down $(0, 1)$

	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
Sign of f''	+	0	-	0	+
Concavity	up	IP	down	IP	up

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1. $y = \frac{1}{3}x^3 + x^2 - 3x + 2$

$$y' = x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x = -3 \quad x = 1$$

inc: $(-\infty, -3) \cup (1, \infty)$
 dec: $(-3, 1)$

	$(-\infty, -3)$	-3	$(-3, 1)$	1	$(1, \infty)$
sign of f'	+	0	-	0	+
inc/dec	inc	CP	dec	CP	inc

$$y'' = 2x + 2 = 0$$

$$x = -1 \quad \text{IP}$$

C. up $(-1, \infty)$
 C. down $(-\infty, -1)$

	$(-\infty, -1)$	-1	$(-1, \infty)$
Sign of f''	-	0	+
Conc	down	IP	up

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2.

a. $f'(x) = 0 \quad x = -1.5, 1.5$
 $f'(x) > 0 \quad (-\infty, -1.5) \cup (1.5, \infty)$
 $f'(x) < 0 \quad (-1.5, 1.5)$

b. $f''(x) = 0 \quad x = 0$
 $f''(x) > 0 \quad (0, \infty)$
 $f''(x) < 0 \quad (-\infty, 0)$

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4. a. when $f(x)$ is inc, $f'(x)$ is pos
 inc: $(-\infty, a) \cup (e, \infty)$
 when $f(x)$ is dec, $f'(x)$ is neg
 dec: $(a, c) \cup (c, e)$

b. when $f(x)$ has max/min, $f'(x) = 0$
 $x = a \quad x = c \quad x = e$

c. $f''(x)$ is slope of $f'(x)$
 up: $(b, c) \cup (d, \infty)$ * where f' is inc.
 down: $(-\infty, b) \cup (c, d)$ * where f' is dec.

d. I.P. where $f''(x) = 0$, where f' has max/min
 $x = b \quad x = d \quad x = c$

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