

5.1 Fundamental Identities

Obj: 1. Use identities to simplify trig expressions.
2. Solve trig. equations.

Reciprocal Identities:

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

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Quotient Identities:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$* \tan^2 \theta + 1 = \sec^2 \theta$$

$$* \tan^2 \theta = \sec^2 \theta - 1$$

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Cofunction Identities:

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta \quad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta \quad \csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta \quad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$$

Odd-Even Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x$$

$$\cot(-x) = -\cot x$$

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Simplify to a single trig func:

$$\sin^3 x + \sin x \cos^2 x$$

$$\sin x (\sin^2 x + \cos^2 x)$$

$$\sin x (1)$$

$$\sin x$$

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Simplify to a constant:

$$\cot x \cdot \tan x \quad \text{OR}$$

$$\frac{\cancel{\cos x}}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}}$$

$$\frac{1}{\cancel{\tan x}} \cdot \frac{\cancel{\tan x}}{1}$$

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Simplify:

$$\frac{(\sec x + 1)(\sec x - 1)}{\sin^2 x}$$

$$\frac{\sec^2 x - 1}{\sin^2 x}$$

$$\frac{\tan^2 x}{\sin^2 x} = \frac{\cancel{\sin^2 x}}{\cos^2 x} \cdot \frac{1}{\cancel{\sin^2 x}} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

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$$\frac{1 + \tan x}{1 + \cot x}$$

$$\frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}}$$

$$\frac{\cos x + \sin x}{\sin x + \cos x} \cdot \frac{\sin x}{\sin x + \cos x}$$

$$\frac{\sin x}{\cos x}$$

$$\tan x$$

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$$\frac{\cos^2 x}{\cos x(1 - \sin x)} - \frac{\sin x(1 - \sin x)}{\cos x(1 - \sin x)}$$

$$\frac{\cos^2 x - \sin x + \sin^2 x}{\cos x(1 - \sin x)}$$

$$\frac{1 - \sin x}{\cos x(1 - \sin x)} = \frac{1}{\cos x} = \sec x$$

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$$\frac{\cancel{\cos x} \sec x}{\cancel{\cos x} \sin x} - \left(\frac{\sin x}{\cos x} \right) \frac{\sin x}{\sin x}$$

$$\frac{\cos x \sec x - \sin^2 x}{\cos x \sin x}$$

$$\frac{\cancel{\cos x} \frac{1}{\cancel{\cos x}} - \sin^2 x}{\cos x \sin x}$$

$$\frac{1 - \sin^2 x}{\cos x \sin x} = \frac{\cos^2 x}{\cancel{\cos x} \sin x} = \frac{\cos x}{\sin x} = \cot x$$

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