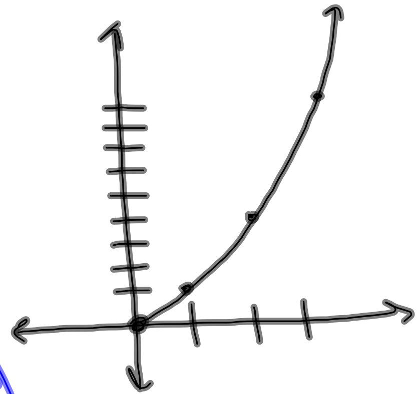
5.3.2

average value:
average y value

$f(x)=x^2 \quad [0,3]$

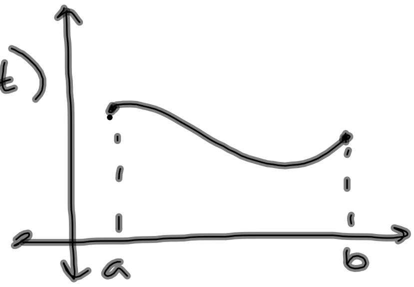


$$\frac{\int_0^3 x^2 dx}{3-0}$$

$$\frac{\frac{1}{3}x^3\Big|_0^3}{3} = \frac{\frac{1}{3}(3^3-0^3)}{3} = 3$$

Jan 20-1:13 PM

$$\frac{\int_a^b v(t)dt}{b-a}$$



$$av(f) = \frac{\int_a^b f(x)dx}{b-a}$$

YES!

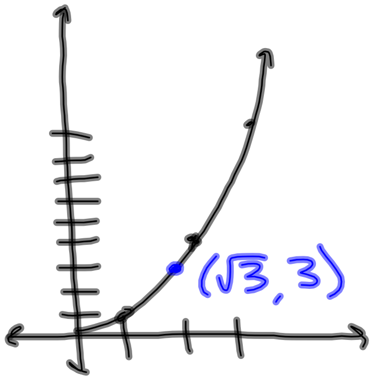
Jan 20-1:19 PM

$f(x)=x^2 \quad [0,3]$

$av(f) = 3$

$x^2 = 3$

$x = \sqrt{3}$



Continuity
MVT for Integrals

Jan 20-1:21 PM

If f is integrable on [a,b] its average (mean) value on [a,b] is:

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

*RS 27

Jan 20-1:24 PM

1. $f(x) = -2x^3 \quad [1,3]$

$$av(f) = \frac{1}{3-1}\int_1^3 -2x^3\,dx$$
$$= \frac{1}{2}\left(-\frac{1}{2}x^4\right)\Big|_1^3 = -\frac{1}{4}(3^4 - 1^4)$$
$$= -\frac{1}{4}(80) = -20$$

$$-2x^3 = -20$$
$$x^3 = 10$$
$$x = \sqrt[3]{10}$$

Jan 20-1:28 PM

2. $f(x) = (x+2)^2 \quad [-1, 1]$

$$av(f) = \frac{1}{1+1}\int_{-1}^{1} (x^2 + 4x + 4)\,dx$$
$$= \frac{1}{2}\left(\frac{1}{3}x^3 + 2x^2 + 4x\right)\Big|_{-1}^{1}$$
$$= \frac{1}{2}\left(\frac{1}{3} + 2 + 4 - \left(-\frac{1}{3} + 2 - 4\right)\right)$$
$$= \frac{1}{2}\left(\frac{19}{3} - \left(-\frac{7}{3}\right)\right) = \frac{1}{2}\left(\frac{26}{3}\right) = \frac{13}{3}$$

$$(x+2)^2 = \frac{13}{3}$$
$$x + 2 = \sqrt{\frac{13}{3}}$$
$$x = -2 + \sqrt{\frac{13}{3}}$$

Jan 20-1:33 PM

3. $g(x) = x^3 + 1 \quad [1, 4]$

$$av(g) = \frac{1}{4-1}\int_1^4 (x^3+1)\,dx$$

$$= \frac{1}{3}\left(\frac{1}{4}x^4 + x\right)\Big|_1^4$$

$$= \frac{1}{3}\left(\frac{1}{4}\cdot 4^4 + 4 - \left(\frac{1}{4}+1\right)\right)$$

$$= \frac{1}{3}\left(68 - \frac{5}{4}\right) = \frac{1}{3}\left(\frac{272}{4} - \frac{5}{4}\right) = \frac{1}{3}\left(\frac{267}{4}\right)$$

$$= \frac{89}{4}$$

$$x^3 + 1 = \frac{89}{4}$$

$$x^3 = \frac{85}{4}$$

$$x = \sqrt[3]{\frac{85}{4}}$$

Jan 20-1:45 PM

4. $h(x) = e^{2x} \quad [0, \ln 2]$

$$av(h) = \frac{1}{\ln 2 - 0}\int_0^{\ln 2} e^{2x}\,dx$$

$$= \frac{1}{\ln 2}\left(\frac{1}{2}e^{2x}\right)\Big|_0^{\ln 2}$$

$$= \frac{1}{2\ln 2}\left(e^{2\ln 2} - e^0\right) = \frac{1}{2\ln 2}(4-1)$$

$$e^{\ln 2^2}$$

$$e^{\ln 4}$$

$$= \frac{3}{2\ln 2}$$

$$\ln e^{2x} = \ln \frac{3}{2\ln 2}$$

$$\frac{2x\,\cancel{\ln e}}{2} = \frac{\ln\left(\frac{3}{2\ln 2}\right)}{2}$$

$$x = \frac{\ln\left(\frac{3}{2\ln 2}\right)}{2}$$

Jan 20-1:48 PM

MVT Deriv

1. Average Rate of Change $\frac{f(b)-f(a)}{b-a}$
2. Deriv $f'(x)$
3. Set them =
4. Solve

MVT Integrals

1. Average Value of Fnc $av(f)$
2. $f(x)$
3. Set them =
4. Solve

Jan 20-1:52 PM