5.4

Fundamental Thm of Calculus

Part I:

$$\frac{d}{dx}\int_c^x f(t)\,dt = f(x)$$

*RS 29

$$y=\int_c^x f(t)\,dt = F(t)\Big|_c^x = F(x)-F(c)$$

$$\frac{dy}{dx}=f(x)$$

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1. $y=\int_\pi^x \cos t\,dt = \sin t\Big|_\pi^x = \sin x - \sin\pi$

$$\frac{dy}{dx}=\cos x$$

2. $y=\int_{-1}^x (t^2+t-1)\,dt$

$$\frac{dy}{dx}=x^2+x-1$$

3. $y=\int_4^{x^2} e^t\,dt = e^t\Big|_4^{x^2} = e^{(x^2)}-e^4$

$$\frac{dy}{dx}=e^{x^2}(2x)$$

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4. $y=\int_{2x}^{x^2} \sin t\,dt = -\cos t\Big|_{2x}^{x^2}$

$= -(\cos x^2 - \cos 2x)$

$= -\cos(x^2) + \cos(2x)$

$\frac{dy}{dx} = \sin x^2(2x) - \sin 2x(2)$

5. $y=\int_{e^x}^{x^2} \ln t\,dt$

$\frac{dy}{dx} = \ln x^2 \cdot 2x - \underline{\ln e^x} \cdot e^x$

$2x\ln x^2 - xe^x$

6. $y=\int_{x}^{3x} \sin t\,dt$

$\frac{dy}{dx} = 3\sin 3x - \sin x \cdot 1$

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Part II

$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

6. $\int_1^4 1x^{\frac{2}{3}}dx$

$\frac{1}{\frac{5}{3}}x^{\frac{5}{3}}$

$\frac{3}{5}x^{\frac{5}{3}}\Big|_1^4 = \frac{3}{5}(4^{\frac{5}{3}} - 1)$

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7. $\int_0^2 \frac{\sqrt{x}+3}{\sqrt{x}}\,dx$

Separate the fraction

$$\int_0^2 \left(\frac{\sqrt{x}}{\sqrt{x}}+\frac{3}{\sqrt{x}}\right)dx$$

$$\int_0^2 \left(1+3x^{-\frac{1}{2}}\right)dx$$

$$\left(x+\frac{3}{\frac{1}{2}}x^{\frac{1}{2}}\right)\Big|_0^2$$

$$\left(x+6x^{\frac{1}{2}}\right)\Big|_0^2 = 2+6\sqrt{2}-0 = 2+6\sqrt{2}$$

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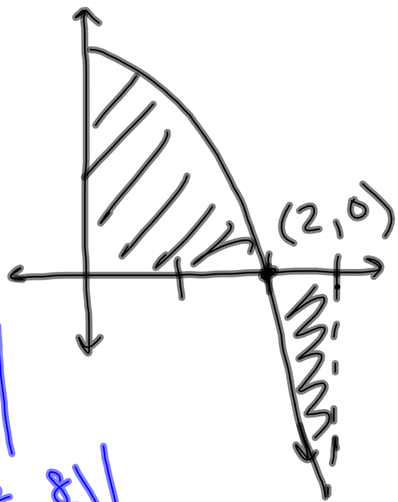
Total area

8. $y = 4-x^2 \quad [0,3]$



$$\int_0^2 (4-x^2)\,dx + \left|\int_2^3 (4-x^2)\,dx\right|$$

$$\left(4x-\frac{1}{3}x^3\right)\Big|_0^2 + \left|\left(4x-\frac{1}{3}x^3\right)\Big]_2^3\right|$$

$$\left(8-\frac{8}{3}\right)-0+\left|12-\frac{27}{3}-\left(8-\frac{8}{3}\right)\right|$$

$$\frac{16}{3}+\left|3-\frac{16}{3}\right| = \frac{16}{3}+\left|-\frac{7}{3}\right| = \frac{23}{3}$$

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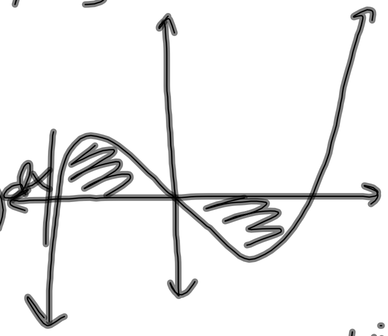
9. $y = x^3 - 4x \quad [-2, 2]$

$$\int_{-2}^{0}(x^3-4x)\,dx + \left|\int_0^2 (x^3-4x)\,dx\right|$$



symmetric

OR

$$2\left|\int_0^2 (x^3-4x)\,dx\right|$$

$$2\left|\left(\tfrac{1}{4}x^4-2x^2\right)\Big|_0^2\right|$$

$$2\left|4-8-0\right| = 8$$

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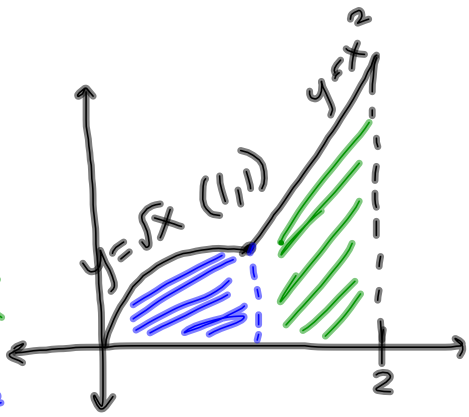
$$\int_0^1 \overset{x^{\frac{1}{2}}}{\sqrt{x}}\,dx + \int_1^2 x^2\,dx$$

$$\tfrac{2}{3}x^{\frac{3}{2}}\Big|_0^1 + \tfrac{1}{3}x^3\Big|_1^2$$

$$\tfrac{2}{3} - 0 + \tfrac{8}{3} - \tfrac{1}{3} = \tfrac{9}{3} = 3$$



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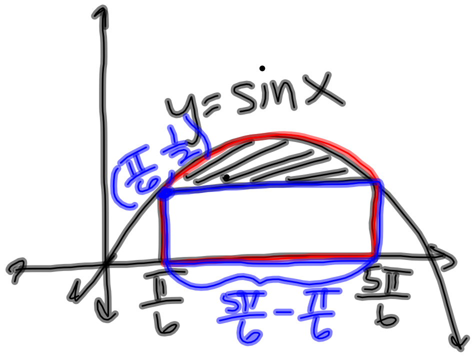

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin x\,dx - \frac{\pi}{3}$$

Rectangle: $\frac{2\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{3}$

$$-\cos x \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \frac{\pi}{3}$$

$$-\left(\cos\frac{5\pi}{6} - \cos\frac{\pi}{6}\right) - \frac{\pi}{3}$$

$$-\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) - \frac{\pi}{3}$$

$$-(-\sqrt{3}) - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3}$$

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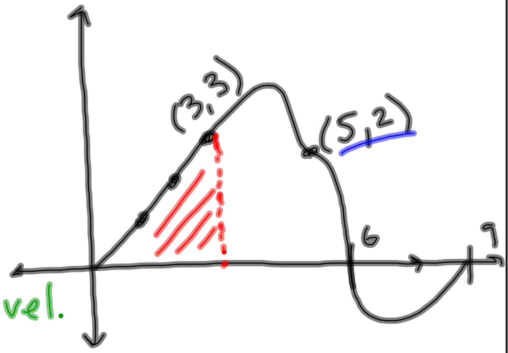
position
$s = \int_0^t f(x)\,dx$



Velocity

a. 2 m/s

b. accel: slope of vel.
negative

c. $\int_0^3 f(x)\,dx = \frac{1}{2}(3)(3) = 4.5$ m

d. t=6 most pos. area

e. t=4,7

f. away: (0,6) toward: (6,9)

g. positive

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