6.2.2

1. $\int_{1}^{2} \frac{2x}{(x^2+3)^2}\,dx$ $\quad u = x^2+3 \quad du = 2x\,dx$ $\quad u(1) = 1^2+3 = 4 \quad u(2) = 2^2+3 = 7$

$$\int_4^7 \frac{du}{u^2} = \int_4^7 1u^{-2}\,du = -u^{-1}\Big|_4^7$$

Change Bounds!

$$= -\left(\frac{1}{7} - \frac{1}{4}\right)$$

$$= -\left(\frac{4}{28} - \frac{7}{28}\right) = \frac{3}{28}$$

2. $\int_e^{e^4} \frac{2}{x\ln x}\,dx$ $\quad u = \ln x \quad 2du = \frac{2}{x}dx$ $\quad u(e) = \ln e = 1 \quad u(e^4) = \ln e^4 = 4$

$$2\int_1^4 \frac{du}{u} = 2\int_1^4 \frac{1}{u}\,du$$

$$2\ln u\Big|_1^4 = 2(\ln 4 - \cancel{\ln 1}^{0}) = 2\ln 4 \text{ OR } \ln 16$$

Feb 12-12:52 PM

3. $\int_0^{\pi} 4\sin x\cos^3 x\,dx$ $\quad (\cos x)^2$ $\quad u = \cos x \quad -4du = 4\sin x\,dx$

$$-4\int_1^{-1} u^2\,du = 4\int_{-1}^{1} u^2\,du = 4\left(\frac{1}{3}u^3\right)\Big|_{-1}^{1}$$

$$= \frac{4}{3}\left(1^3 - (-1)^3\right) = \frac{4}{3}(2) = \frac{8}{3}$$

4. $\int_{\pi/6}^{\pi/3} -\cot x\csc^2 x\,dx$

Option 1

$u = \csc x$

$du = -\csc x\cot x\,dx$

$$\int_2^{2/\sqrt{3}} u\,du = \frac{1}{2}u^2\Big|_2^{2/\sqrt{3}}$$

$$\frac{1}{2}\left(\left(\frac{2}{\sqrt{3}}\right)^2 - 2^2\right)$$

$$\frac{1}{2}\left(\frac{4}{3} - 4\right)$$

$$\frac{1}{2}\left(\frac{4}{3} - \frac{12}{3}\right) = \frac{1}{2}\left(-\frac{8}{3}\right)$$

$$= -\frac{4}{3}$$

Option 2

$u = \cot x$

$du = -\csc^2 x\,dx$

$$\int_{\sqrt{3}}^{\sqrt{3}/3} u\,du = \frac{1}{2}u^2\Big|_{\sqrt{3}}^{\sqrt{3}/3}$$

$$\frac{1}{2}\left(\left(\frac{\sqrt{3}}{3}\right)^2 - (\sqrt{3})^2\right)$$

$$= \frac{1}{2}\left(\frac{3}{9} - 3\right) = \frac{1}{2}\left(\frac{1}{3} - \frac{9}{3}\right)$$

$$= \frac{1}{2}\left(-\frac{8}{3}\right) = -\frac{4}{3}$$

Feb 12-1:31 PM

5. $\int_0^6 \frac{20}{x^2+36}dx = 20\int_0^6 \frac{dx}{36\left(\frac{x^2}{36}+1\right)} = \frac{20}{36}\int_0^6 \frac{dx}{\left(\frac{x}{6}\right)^2+1}$ $u=\frac{x}{6}$ $6du=\cancel{\frac{1}{6}}dx$

$\frac{20}{36}(6)\int_0^1 \frac{du}{u^2+1} = \frac{10}{3}\left(\tan^{-1}u\right)\Big|_0^1$

$= \frac{10}{3}\left(\tan^{-1}1 - \tan^{-1}0\right) = \frac{10}{3}\left(\frac{\pi}{4}-0\right) = \frac{10\pi}{12} = \frac{5\pi}{6}$

6. $\int_0^1 r\sqrt{1-r^2}\,dr$ $u=1-r^2$ $-\frac{1}{2}du = \frac{-2rdr}{-2}$

$-\frac{1}{2}\int_1^0 \sqrt{u}\,du = \frac{1}{2}\int_0^1 \sqrt{u}\,du = \cancel{\frac{1}{2}}\left(\cancel{\frac{2}{3}}u^{\frac{3}{2}}\right)\Big|_0^1$

$\frac{1}{3}\left(1^{\frac{3}{2}} - 0^{\frac{3}{2}}\right) = \frac{1}{3}$

Feb 12-1:39 PM

7. $\int_{-1}^1 \frac{5x}{(4+x^2)^2}dx$ $u=4+x^2$ $\frac{1}{2}du = \frac{2xdx}{2}$

$\frac{5}{2}\int_5^5 \frac{du}{u^2} = 0$

8. $\int_2^5 \frac{dx}{2x-3}$ $u=2x-3$ $\frac{1}{2}du = \frac{2dx}{2}$

$\frac{1}{2}\int_1^7 \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_1^7 = \frac{1}{2}(\ln 7 - \cancel{\ln 1})$

$\frac{1}{2}\int_1^7 \frac{1}{u}du$ $= \frac{1}{2}\ln 7$

Feb 12-1:47 PM

9. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ $\quad u=\sqrt{x}$

$2du = \cancel{2}\frac{1}{\cancel{2}\sqrt{x}}dx$

$\int \sin\sqrt{x}\left(\frac{1}{\sqrt{x}}dx\right)$ u $2du$

$2\int \sin u\,du = -2\cos u + C = -2\cos\sqrt{x} + C$

10. $\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_0^{13} \frac{dx}{(1+2x)^{\frac{2}{3}}} = \int_0^{13} (1+2x)^{-\frac{2}{3}}\,dx$

$u=1+2x$

$\frac{1}{2}du = \frac{\cancel{2}dx}{\cancel{2}}$

$\frac{1}{2}\int_1^{27} 1u^{-\frac{2}{3}}du = \frac{1}{2}\left(3u^{\frac{1}{3}}\right)\Big|_1^{27}$

$\frac{3}{2}\left(27^{\frac{1}{3}} - 1^{\frac{1}{3}}\right) = \frac{3}{2}(3-1) = 3$

Feb 12-1:57 PM