75. $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

76. $\int e^x dx = e^x + C$

77. $\int a^x dx = \frac{1}{\ln a} a^x + C$

78. $\int \frac{1}{x} dx = \ln|x| + C$

79. $\int \sin x dx = -\cos x + C$

80. $\int \cos x dx = \sin x + C$

81. $\int \sec^2 x dx = \tan x + C$

82. $\int \csc^2 x dx = -\cot x + C$

83. $\int \sec x \tan x dx = \sec x + C$

84. $\int \csc x \cot x dx = -\csc x + C$

85. $\int \tan x dx = -\ln|\cos x| + C$

86. $\int \cot x dx = \ln|\sin x| + C$

87. $\int \sec x dx = \ln|\sec x + \tan x| + C$

88. $\int \csc x dx = -\ln|\csc x + \cot x| + C$

89. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

90. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

91. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

Feb 13-9:03 AM

6.4.1

$\cancel{1}\,\frac{dy}{\cancel{dx}} = \frac{2x}{y}\,dx \qquad y=4 \quad x=3$

$dy = \frac{2x\,dx}{y}$

$\int y\,dy = \int 2x\,dx$

$\frac{1}{2}y^2 = x^2 + C$

$\frac{1}{2}(4)^2 = 3^2 + C$

$C = -1$

$\frac{1}{2}y^2 = x^2 - 1$

$y^2 = 2x^2 - 2$

$y = \sqrt{2x^2 - 2}$

Steps

1. Separate variables
2. Integrate both sides
3. Substitute to find C
4. Plug in C. Solve for y.

2. $\frac{dy}{dx} = 4xy \quad y = e^4 \quad x = 1$

$\int \frac{dy}{y} = \int 4x\,dx$

$\ln|y| = 2x^2 + C$

$\ln e^4 = 2(1)^2 + C$

$4 = 2 + C$

$C = 2$

$\ln|y| = 2x^2 + 2$

changing to exponential

$e^{2x^2+2} = y$

3. $\frac{dy}{dx} = e^{x+y}$ $y=4$ $x=0$

$\frac{dy}{dx} = e^x \cdot e^y$

$\int \frac{dy}{e^y} = \int e^x dx$

$\int e^{-y} dy = \int e^x dx$

$-e^{-y} = e^x + C$

$-e^{-4} = e^0 + C$

$C = -e^{-4} - 1$

$-e^{-y} = e^x - e^{-4} - 1$

$\ln e^{-y} = \ln(e^x + e^{-4} + 1)$

$-y = \ln(-e^x + e^{-4} + 1)$

$y = -\ln(-e^x + e^{-4} + 1)$

4. $\frac{dy}{dx} = 3y \quad y=10 \quad x=0$

$\int \frac{dy}{y} = \int 3\,dx$

$\ln|y| = 3x + C$

$\ln 10 = C$

$\ln|y| = 3x + \ln 10$

$e^{3x+\ln 10} = y$

$y = e^{3x} \cdot e^{\ln 10}$

$y = 10e^{3x}$

FR 2005 #6

$\frac{dy}{dx} = -\frac{xy^2}{2}dx \quad f(-1) = 2$ (x, y)

c. $\int \frac{dy}{y^2} = \int -\frac{x}{2}dx$

$\int y^{-2}dy = -\frac{1}{2}\int x\,dx$

$-y^{-1} = -\frac{1}{2}\left(\frac{1}{2}x^2\right) + C$

$-\frac{1}{y} = -\frac{1}{4}x^2 + C$

$-\frac{1}{2} = -\frac{1}{4}(-1)^2 + C$

$C = -\frac{1}{4}$

$-\frac{1}{y} = -\frac{1}{4}x^2 - \frac{1}{4}$

$\frac{1}{y} = \frac{x^2}{4} + \frac{1}{4}$

$\frac{1}{y} = \frac{x^2+1}{4}$

$y = \frac{4}{x^2+1}$

b. $y - y_1 = m(x - x_1)$ $\quad (-1, 2)$

$m: \frac{dy}{dx} = -\frac{xy^2}{2}\Big|_{(-1,2)} = 2$

$y - 2 = 2(x+1)$