6.4.2

$$y = y_0 e^{kt}$$

exponential change:
rate of change is proportional to the amount present.

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{dt} = ky \qquad y = y_0 \quad t = 0$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| = kt + C$$

$$\ln y_0 = C$$

$$\ln y = kt + \ln y_0$$

$$e^{kt + \ln y_0} = y$$

$$e^{kt} \cdot \cancel{e^{\ln}} y_0 = y$$

$$y = y_0 e^{kt}$$

Calc: $\frac{dy}{dt} = ky$

Algebra: $y = y_0 e^{kt}$

1. $\frac{dy}{dt} = -2.1y$ $\quad y_0 = 100$

$k$

$y = y_0 e^{kt}$

$y = 100e^{-2.1t}$

2. $y = y_0 e^{kt}$

$(0, \underset{y_0}{1.1})$ $(\underset{t}{3}, \underset{y}{3})$

$y = 1.1e^{kt}$

$\frac{3}{1.1} = \frac{1.1e^{3k}}{1.1}$

$\ln \frac{3}{1.1} = \ln e^{3k}$

$\frac{\ln \frac{3}{1.1}}{3} = \frac{3k}{3}$

$k = \frac{\ln \frac{3}{1.1}}{3} \approx .3344$

$y = 1.1e^{.3344t}$

3. "rate proportional to the amt. present"

$$y = y_0 e^{kt}$$

$$3000 = y_0 e^{3K}$$

$$\frac{5000}{e^{7K}} = \frac{y_0 e^{7K}}{e^{7K}}$$

$$y_0 = \frac{5000}{e^{7K}}$$

$$3000 = \frac{5000}{e^{7K}} e^{3K}$$

$$3000 = 5000e^{-4K}$$

$$\ln\frac{3}{5} = \ln e^{-4K}$$

$$\frac{\ln\frac{3}{5}}{-4} = \frac{-4K}{-4}$$

$$K \approx .1277\ldots\ldots$$

$$y_0 = \frac{5000}{e^{7K}}$$

$\approx 2045.195$ bacteria

$y = 2045.195e^{K(10)} \approx 7334$ bacteria

4. $y = y_0 e^{-.06t}$

$\frac{\frac{1}{2}\cancel{y_0}}{\cancel{y_0}} = \frac{\cancel{y_0} e^{-.06t}}{\cancel{y_0}}$

$\frac{1}{2} = e^{-.06t}$

$t = \frac{\ln \frac{1}{2}}{-.06} \approx 11.552$

$\frac{.2\cancel{y_0}}{\cancel{y_0}} = \frac{\cancel{y_0} e^{-.06t}}{\cancel{y_0}}$

$.2 = e^{-.06t}$

$t = \frac{\ln .2}{-.06}$

$\approx 26.824$

5. $\frac{dy}{dt} = -1.5y$ 10,000

$k$ $y_0$

$y = y_0 e^{kt}$

$y = 10000e^{-1.5t}$

$100 = 10000e^{-1.5t}$

$.01 = e^{-1.5t}$

$t = \frac{\ln .01}{-1.5} \approx 3.07$ sec.

temp of surr.

$$T - T_s = (T_0 - T_s)\, e^{-kt}$$

final temp   init. temp.   time

$T_0 = 200$

$T_s = 70$

$T = 100$

$100 - 70 = (200 - 70)\, e^{-kt}$

$30 = 130 e^{-kt}$

$\ln \frac{3}{13} = \cancel{\ln} e^{-kt}$

$\ln \frac{3}{13} = -kt$

$t = \dfrac{\ln \frac{3}{13}}{-k} \approx 5.589$ min

$t = 1 \qquad T = 170$

$170 - 70 = (200 - 70)\, e^{-k}$

$100 = 130 e^{-k}$

$\frac{10}{13} = e^{-k}$

$\ln \frac{10}{13} = -k$

$k = -\ln \frac{10}{13}$

$\approx .2623 \ldots\ldots$