7.2 Matrix Algebra

Obj: 1. Find sums, differences, products & inverses of matrices.

$$\begin{array}{ll} & \text{2nd column} \\ \text{Matrix:} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

1st row

2nd row

order: $m \times n$

Mar 15-9:28 AM

Find the order:

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix} \quad \boxed{2 \times 3}$$

2 rows 3 columns

Find $a_{23} = 4$

Mar 15-9:57 AM

Find the order:

$$\begin{bmatrix} 1 & -1 \\ 0 & 4 \\ 2 & -1 \\ 3 & 2 \end{bmatrix} \quad 4\times2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad 3\times3$$

Square matrix

Mar 15-9:59 AM

Addition / Subtraction: Same order

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 2 \\ 4 & -3 & -1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 6 & -3 & 0 \end{bmatrix} \qquad A-B = \begin{bmatrix} -3 & -1 & 2 \\ 5 & 1 & -3 \\ -2 & 3 & 2 \end{bmatrix}$$

Mar 15-10:00 AM

Scalar Multiplication

Constant

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} \quad k = 3$$

Find $kA = \begin{bmatrix} 6 & 9 \\ -3 & 15 \end{bmatrix}$

Mar 15-10:04 AM

Matrix Multiplication

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -4 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$$

2×3 possible 3×2

Find AB.

answer: 2×2

$$\begin{bmatrix} 2+0+-3 & -8+2+0 \\ 0+0+2 & 0+2+0 \end{bmatrix} = \begin{bmatrix} -1 & -6 \\ 2 & 2 \end{bmatrix}$$

$AB \neq BA$

Mar 15-10:06 AM

$$A=\begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 2 \end{bmatrix} \quad B=\begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix}$$

$2\times\underline{3}$ $\underline{2}\times 2$

Find AB not possible

BA

2×2 2×3 – possible

Mar 15-10:13 AM

$$A=\begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & -3 \end{bmatrix} \quad B=\begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$2\times\underline{3}$ $\underline{3}\times 2$

Find AB :

2×2

$$\begin{bmatrix} 2+0+0 & 4+0+-2 \\ 1+-12+0 & 2+4+6 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -11 & 12 \end{bmatrix}$$

Mar 15-10:16 AM

Transpose: switch rows & columns

$$A = \begin{bmatrix} 5 & 8 & 7 \\ 6 & 6 & 7 \\ 4 & 3 & 3 \end{bmatrix} \qquad A^T = \begin{bmatrix} 5 & 6 & 4 \\ 8 & 6 & 3 \\ 7 & 7 & 3 \end{bmatrix}$$

Mar 15-10:21 AM

Identity Matrix: square matrix

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If $AB = BA = I_n$

then B is the inverse of A

$B = A^{-1}$

Mar 15-10:23 AM

Prove $A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

are inverses.

$AB = BA = I_2$

$AB = \begin{bmatrix} \underline{3-2} & \underline{6+-6} \\ \underline{-1+1} & \underline{-2+3} \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \underline{3+-2} & \underline{-2+2} \\ \underline{3+-3} & \underline{-2+3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

Mar 15-10:27 AM

Not all matrices have inverses!

<u>Determinant</u>

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det A = ad - bc$

If $\det A \neq 0$ then matrix A has an inverse.

<u>Inverse</u>: $A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Mar 15-10:32 AM