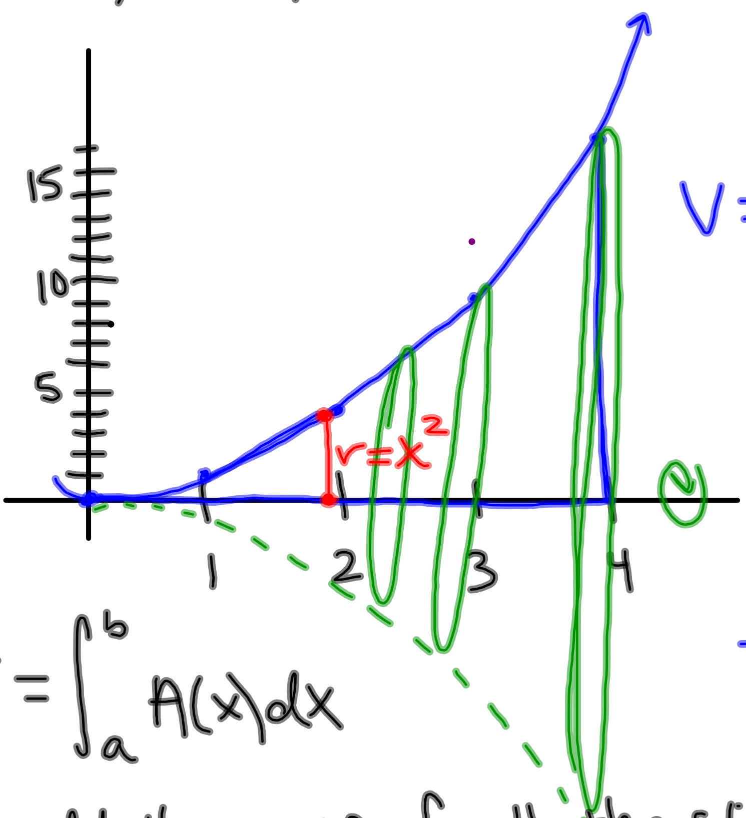
7.3.1

$f(x) = x^2$, x-axis $[0, 4]$



$$V = \int_0^4 \pi r^2 \, dx$$

$$\pi \int_0^4 (x^2)^2 \, dx$$

$$\pi \int_0^4 x^4 \, dx$$

$$\pi \left(\frac{1}{5} x^5 \right) \Big|_0^4$$

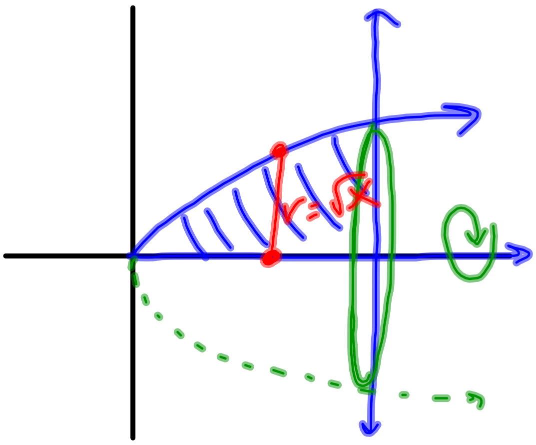
$$\frac{\pi}{5} (4^5 - 0^5)$$

$$\frac{1024\pi}{5}$$

$$V = \int_a^b A(x) \, dx$$

* add the areas of all the slices!

1. $y=\sqrt{x} \quad x=4 \quad y=0$



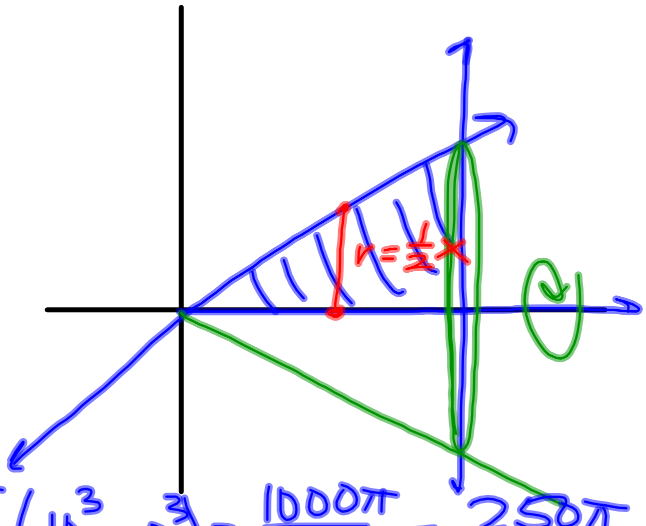
$$V=\pi\int_0^4(\sqrt{x})^2dx$$

$$=\pi\int_0^4 x\,dx$$

$$=\pi\left(\tfrac{1}{2}x^2\right)\Big|_0^4=\frac{\pi}{2}(4^2-0^2)$$

$$=8\pi$$

2. $y=\frac{1}{2}x \quad y=0 \quad x=10$



$$V=\pi\int_0^{10}\left(\frac{1}{2}x\right)^2dx$$

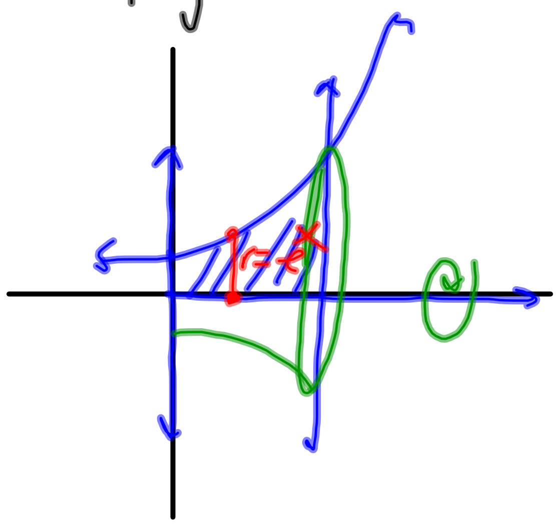
$$\frac{\pi}{4}\int_0^{10}x^2dx$$

$$\frac{\pi}{4}\left(\frac{1}{3}x^3\right)\Big|_0^{10}=\frac{\pi}{12}(10^3-0^3)=\frac{1000\pi}{12}=\frac{250\pi}{3}$$

OR: CONE!

$$V=\frac{1}{3}\pi r^2h=\frac{\pi}{3}(5)^2(10)$$

3. $\underline{y=e^x}$, $x=0$, $x=\ln 4$, $y=0$



$$V=\pi\int_0^{\ln 4}(e^x)^2\,dx$$

$$=\pi\int_0^{\ln 4} e^{2x}\,dx$$

$$\pi\left(\frac{1}{2}e^{2x}\right)\Big|_0^{\ln 4}$$

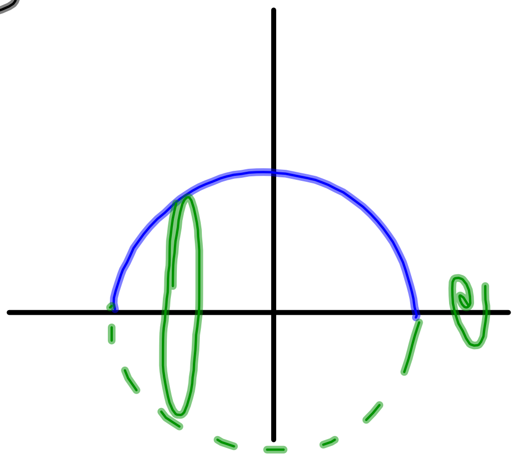
$$\frac{\pi}{2}\left(e^{2\ln 4}-e^{0}\right)=\frac{\pi}{2}(16-1)=\frac{15\pi}{2}$$

4. $y = \sqrt{4-x^2}$, $y = 0$

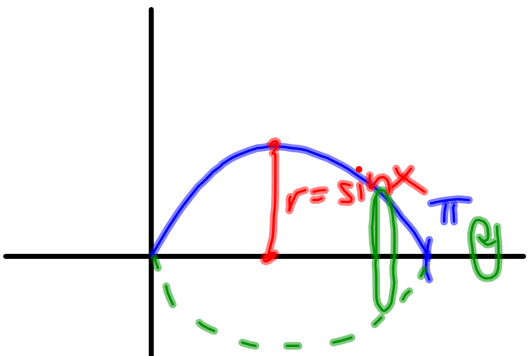
Sphere!

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (2)^3 = \frac{32\pi}{3}$



5. $y = \sin x,\ y = 0 \quad [0, \pi]$



$$V = \pi \int_0^{\pi} (\sin x)^2 \, dx$$

$$= \pi \int_0^{\pi} \sin^2 x \, dx$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$= \frac{\pi}{2} \int_0^{\pi} (1 - \cos 2x) \, dx$$

$$= \frac{\pi}{2}\left(x - \frac{1}{2}\sin 2x\right)\Big|_0^{\pi}$$

$$= \frac{\pi}{2}\left[\left(\pi - \frac{1}{2}\sin 2\pi\right) - \left(0 - \frac{1}{2}\sin 0\right)\right]$$

$$= \frac{\pi}{2}(\pi - 0) = \frac{\pi^2}{2}$$

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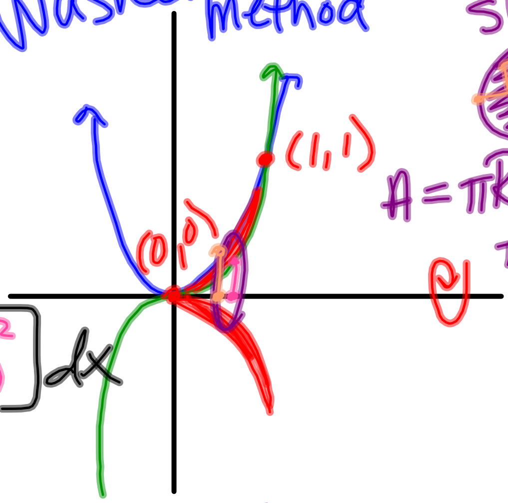
6. $y = x^2$ $\quad$ $y = x^3$

$x^2 = x^3$

Washer method



slice

$A = \pi R^2 - \pi r^2$

$\pi(R^2 - r^2)$

$$V = \pi \int_0^1 \left[(x^2)^2 - (x^3)^2 \right] dx$$

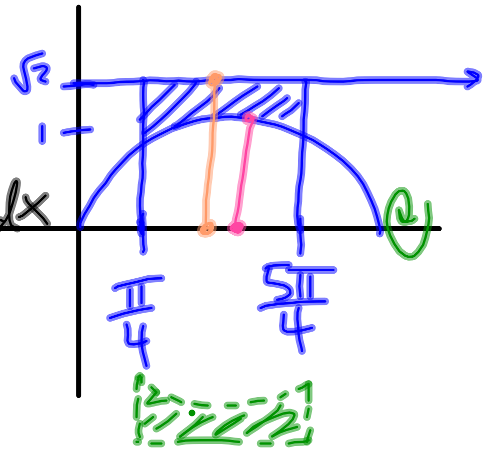
$$= \pi \int_0^1 (x^4 - x^6)\, dx$$

$\vdots$

There's a hole in the middle!

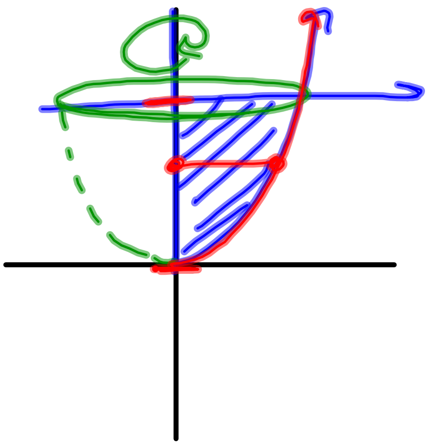
7. $y = \sin x$, $y = \sqrt{2}$, $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$$V = \pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left[(\sqrt{2})^2 - (\sin x)^2 \right] dx$$



8. $y=x^2$, $x=0$, $y=4$

$x=\sqrt{y}$



$V=\pi\int_0^4 (\sqrt{y})^2\,dy$

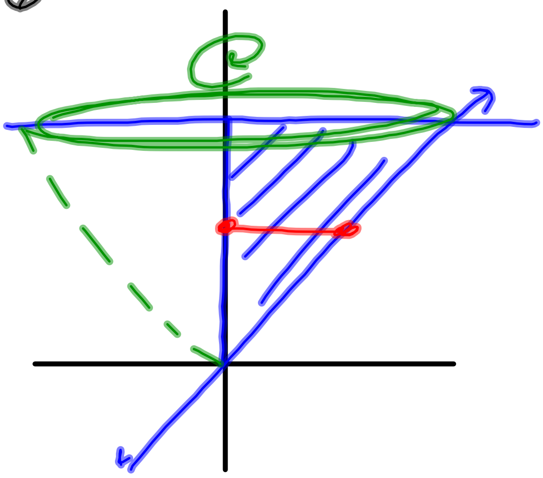
$\pi\int_0^4 y\,dy$

$\vdots$

9. $y=2x$, $x=0$, $y=6$

$x=\frac{1}{2}y$

$$V=\pi\int_0^6\left(\frac{1}{2}y\right)^2dy$$



10. $y = x^2$, $x = 4$, $y = 0$

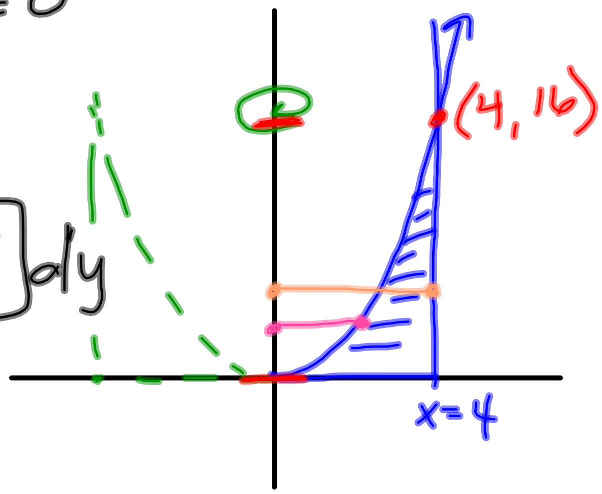
$x = \sqrt{y}$

$$V = \pi \int_0^{16} \left[(4)^2 - (\sqrt{y})^2 \right] dy$$

$$= \pi \int_0^{16} (16 - y)\, dy$$

$\vdots$

$$= 128\pi$$

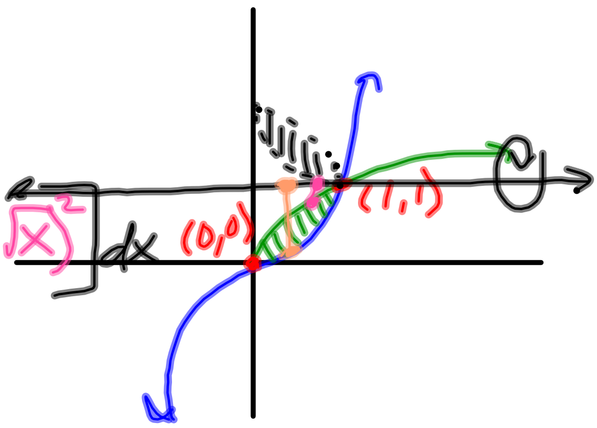


11. $y=x^3$, $y=\sqrt{x}$, about $y=1$

$x^3=\sqrt{x}$

$$V=\pi\int_0^1\left[(1-x^3)^2-(1-\sqrt{x})^2\right]dx$$

Top - Bottom



12. $y=x^3$ $y=\sqrt{x}$ about $y=-3$

$$V=\pi\int_0^1\left[(\sqrt{x}+3)^2-(x^3+3)^2\right]dx$$

⋮