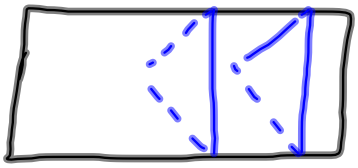
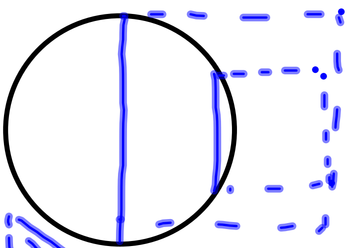
7.3.3

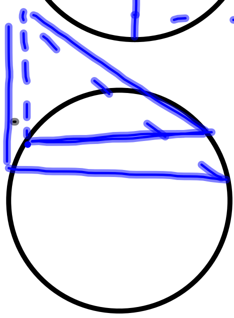
a.



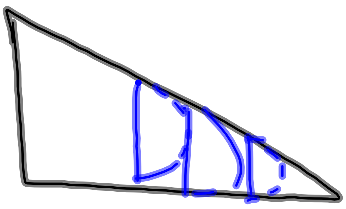
b.



c



e.



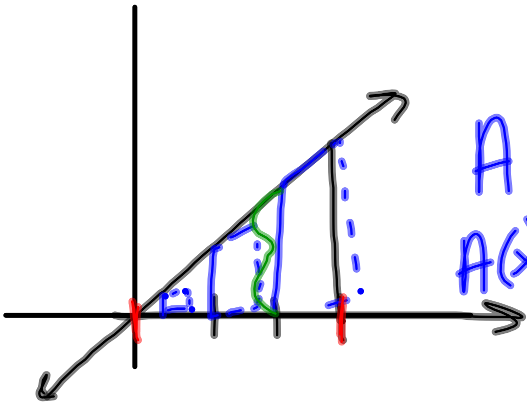
Strategy:

adds up area of each cross section

R.S. #40

$$V = \int_a^b A(x)\,dx$$

1. Base: $y=x$ $y=0$ $[0,3]$



$A = s^2$

$A(x) = (x)^2$

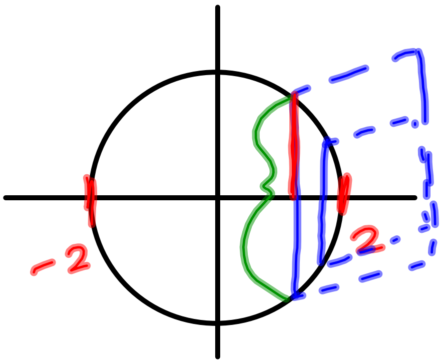
$$V = \int_a^b A(x)\,dx$$

$$V = \int_0^3 x^2\,dx = \frac{1}{3}x^3\Big|_0^3 = \frac{1}{3}(3^3 - 0^3) = 9$$

2. Base: circle rad. 2

$x^2+y^2=4$

$y=\sqrt{4-x^2}$



$A=s^2$

$A(x)=\left(2\sqrt{4-x^2}\right)^2$

$4(4-x^2)$

$$V=\int_{-2}^{2}4(4-x^2)\,dx$$

OR

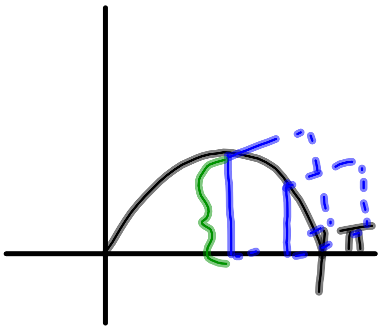
$$V=2\int_{0}^{2}4(4-x^2)\,dx$$

$$8\int_0^2(4-x^2)\,dx=8\left(4x-\tfrac{1}{3}x^3\right)\Big|_0^2$$

$\vdots$

3. Base $y=\sin x$ $y=0$ $[0,\pi]$

A.



$A = s^2$

$A(x) = (\sin x)^2$

$$V = \int_0^{\pi} \sin^2 x\,dx = \int_0^{\pi} \frac{1}{2}(1-\cos 2x)\,dx$$

$$= \frac{1}{2}\int_0^{\pi} (1-\cos(2x))\,dx$$

$-\frac{1}{2}\int \cos u\,du$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$$\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)_0^{\pi} = \frac{1}{2}\left(\pi - \cancel{\frac{1}{2}\sin 2\pi} - 0\right)$$

$$= \frac{\pi}{2}$$

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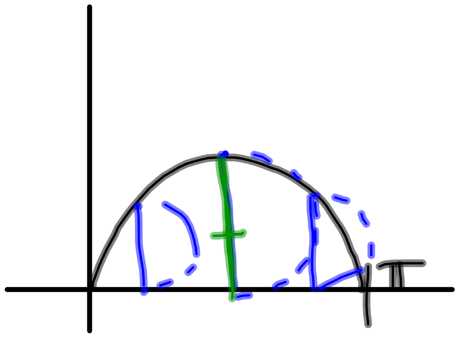
B. $y=\sin x \quad y=0 \quad [0,\pi]$



$A=\frac{1}{2}\pi r^2$

$A(x)=\frac{1}{2}\pi\left(\frac{1}{2}\sin x\right)^2$

$=\frac{\pi}{8}\sin^2 x$

$d=\sin x$

$r=\frac{1}{2}d=\frac{1}{2}\sin x$

$$V=\int_0^{\pi}\frac{\pi}{8}\sin^2 x\,dx$$

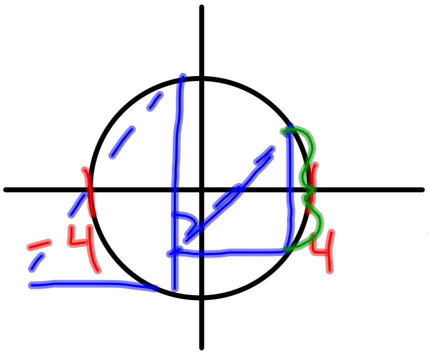
$$\frac{\pi}{8}\int_0^{\pi}\underline{\sin^2 x}\,dx=\frac{\pi}{8}\int_0^{\pi}\frac{1}{2}(1-\cos 2x)\,dx$$

$$=\frac{\pi}{16}\int_0^{\pi}(1-\underline{\cos 2x})\,dx=\frac{\pi}{16}\left(x-\frac{1}{2}\sin 2x\right)\Big|_0^{\pi}$$

$$=\frac{\pi}{16}\left(\pi-\frac{1}{2}\sin 2\pi-0\right)=\frac{\pi^2}{16}$$

4. Base : circle rad. 4 $x^2+y^2=16$

$y=\sqrt{16-x^2}$



$A=\frac{1}{2}bh=\frac{1}{2}b^2$

$A(x)=\frac{1}{2}\left(2\sqrt{16-x^2}\right)^2=\frac{1}{2}\left(4(16-x^2)\right)$

$=2(16-x^2)$

$V=\int_{-4}^{4}$

OR $V=2\int_0^4 2(16-x^2)\,dx$

$4\int_0^4 (16-x^2)\,dx = 4\left(16x-\frac{1}{3}x^3\right)\Big|_0^4$ ...

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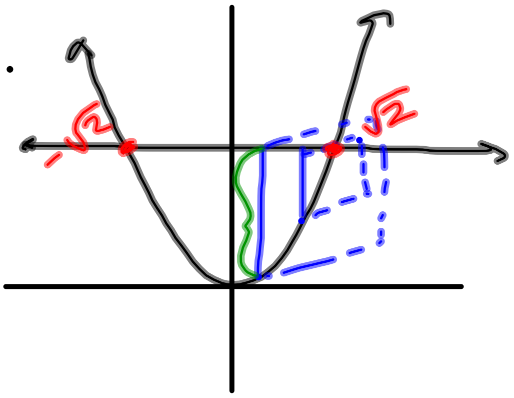
5. Base $y=x^2$, $y=2$

$x^2=2$

$x=\pm\sqrt{2}$

A.



$A=s^2$

$A(x)=(2-x^2)^2$

$$V=\int_{-\sqrt{2}}^{\sqrt{2}}(2-x^2)^2\,dx$$

$(2-x^2)(2-x^2)$

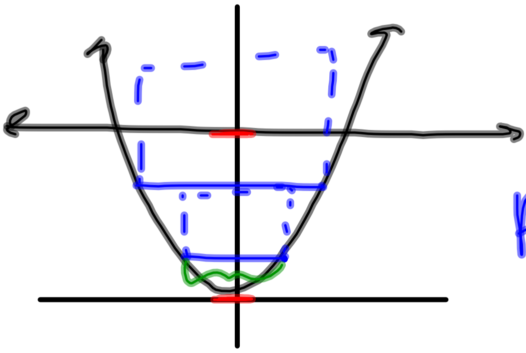
OR $$V=2\int_0^{\sqrt{2}}(2-x^2)^2\,dx$$

$$2\int_0^2(4-4x^2+x^4)\,dx$$

$\vdots$

$x = \sqrt{y}$

B. $y = x^2$ $y = 2$



$A = s^2$

$A(y) = (2\sqrt{y})^2 = 4y$

$V = \int_0^2 4y\,dy$

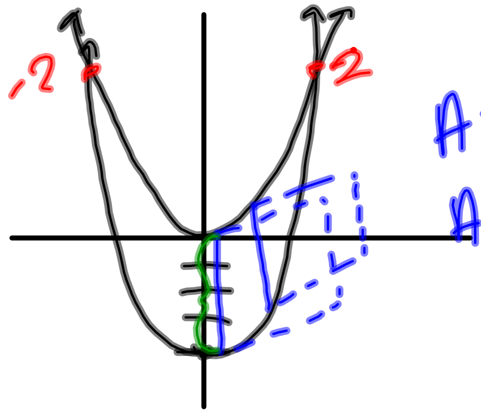
⋮

Mar 11-2:04 PM

6. Base $y=x^2$ $y=2x^2-4$

$x^2=2x^2-4$

$x=\pm 2$



$A=s^2$

$A(x)=(x^2-(2x^2-4))^2$

$=(x^2-2x^2+4)^2=(-x^2+4)^2$

$$V=\int_{-2}^{2}(-x^2+4)^2\,dx$$

OR

$$V=2\int_{0}^{2}(-x^2+4)^2\,dx$$

$(-x^2+4)(-x^2+4)$

⋮