7.2 Cont

A matrix has an inverse iff $\det A \neq 0$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \det A = ad - bc$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find A^{-1} if $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$\det A = 3 \cdot 2 - 1 \cdot 4 = 2$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

Find $\det B$ and B^{-1} if $B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$

$\det B = -10$

$B^{-1} \begin{bmatrix} .1 & .2 & -.5 \\ .5 & 0 & .5 \\ .1 & .2 & .5 \end{bmatrix}$

7.3 Day 1

Multivariate Linear Systems & Row Operations

Obj: 1. Solve systems of linear equations using Gaussian elimination & row echelon form.

Solve:

$$x - 2y + z = 7$$
$$y - 2z = -7$$
$$z = 3$$

$y - 6 = -7$

$y = -1$

$x + 2 + 3 = 7$

$x = 2$

$(2, -1, 3)$

Solve: $3x - y + 2z = -2$

$y + 3z = 3$

$\frac{2z}{2} = \frac{4}{2}$

$z = 2$

$y + 6 = 3$

$y = -3$

$3x + 3 + 4 = -2$

$3x + 7 = -2$

$3x = -9$

$x = -3$

$(-3, -3, 2)$

Gaussian Elimination

Solve: R_1 $1x-2y+z=7$

R_2 ~~$3x-5y+z=14$~~

R_3 $2x-2y-z=3$

$-3R_1+R_2 =$

$$\begin{array}{r} -3\cancel{x}+6y-3z=-21 \\ \cancel{3}x-5y+z=14 \\ \hline y-2z=-7 \end{array}$$

$1x-2y+z=7$

$y-2z=-7$

~~$2x-2y-z=3$~~

$-2R_1+R_3 =$

$$\begin{array}{r} -2\cancel{x}+4y-2z=-14 \\ \cancel{2}x-2y-z=3 \\ \hline 2y-3z=-11 \end{array}$$

$x-2y+z=7$

$1y-2z=-7$

~~$2y-3z=-11$~~

$-2R_2+R_3 =$

$$\begin{array}{r} -2\cancel{y}+4z=14 \\ \cancel{2}y-3z=-11 \\ \hline z=3 \end{array}$$

$x-2y+z=7$

$y-2z=-7$

$z=3$

$(2,-1,3)$

Row Operations & Row Echelon Form

$$\begin{aligned} 1x - 2y + 1z &= 7 \\ 3x - 5y + 1z &= 14 \\ 2x - 2y - 1z &= 3 \end{aligned}$$

augmented matrix

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & \text{constant} \end{array} \\ \left[\begin{array}{cccc} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array}\right] \end{array}$$

$-3R_1 + R_2$

$$\left[\begin{array}{cccc} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 2 & -2 & -1 & 3 \end{array}\right] \qquad \begin{aligned} x - 2y + z &= 7 \\ y - 2z &= -7 \\ 2x - 2y - z &= 3 \end{aligned}$$

$-2R_1 + R_3$

$$\left[\begin{array}{cccc} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 2 & -3 & -11 \end{array}\right] \qquad \begin{aligned} x - 2y + z &= 7 \\ y - 2z &= -7 \\ 2y - 3z &= -11 \end{aligned}$$

$-2R_2 + R_3$

$$\left[\begin{array}{cccc} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right] \qquad \begin{aligned} x - 2y + z &= 7 \\ y - 2z &= -7 \\ z &= 3 \end{aligned}$$

Row Echelon Form

Solve

Solving Systems Using Inverses:

Solve using an inverse: $\begin{cases} 3x - 2y = 0 \\ -x + y = 5 \end{cases}$

matrix equation

$$\overset{A=}{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}} \cdot \overset{X=}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 0 \\ 5 \end{bmatrix}}$$

$A \cdot X = B$

$$X = A^{-1}B = \begin{bmatrix} 10 \\ 15 \end{bmatrix} \begin{matrix} = x \\ = y \end{matrix}$$

Row Echelon Form:

1. Rows of all zeros go on the bottom.
2. 1^{st} entry in a row is 1.
3. Leading ones go diagonally from L to R.

leading ones

$$\begin{bmatrix} \underline{1} & -2 & 1 & 7 \\ 0 & \underline{1} & -2 & -7 \\ 0 & 0 & \underline{1} & 3 \end{bmatrix}$$

Mar 19-10:29 AM