

7.3 Products & Factors of Polynomials

Obj: 1. Multiply & divide polynomials.
2. Factor Thm.

Write $x(16-2x)(12-2x)$ as a polynomial in standard form.

$$\begin{aligned}
 & x(192 - 32x - 24x + 4x^2) \\
 & x(192 - 56x + 4x^2) \\
 & 192x - 56x^2 + 4x^3 \\
 & 4x^3 - 56x^2 + 192x
 \end{aligned}$$

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$$2x^2(x^2+2)(x-3)$$

$$2x^2(x^3 - 3x^2 + 2x - 6)$$

$$2x^5 - 6x^4 + 4x^3 - 12x^2$$

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Factor

$$x^3 - 5x^2 - 6x$$

$$x(x^2 - 5x - 6)$$

$$x(x-6)(x+1)$$

$$(x^3 + 4x^2) + (2x + 8)$$

$$x^2(x+4) + 2(x+4)$$

$$(x+4)(x^2+2)$$

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$$x^3 - 9x$$

$$x(x^2 - 9)$$

$$x(x-3)(x+3)$$

$$(x^3 - x^2) + (2x - 2)$$

$$x^2(x-1) + 2(x-1)$$

$$(x-1)(x^2+2)$$

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Long Division

$$\begin{array}{r}
 327 \leftarrow \text{quotient} \\
 \text{divisor} \rightarrow \underline{12} \overline{) 3924} \leftarrow \text{dividend} \\
 \underline{-36} \\
 32 \\
 \underline{-24} \\
 84 \\
 \underline{-84} \\
 0 \text{ remainder}
 \end{array}$$

$$12 \cdot 327 = 3924$$

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$$(x^3 + 3x^2 + 3x + 2) \div (x^2 + x + 1)$$

$$\begin{array}{r}
 x+2 \\
 \underline{x^2+x+1} \overline{) x^3+3x^2+3x+2} \\
 \underline{-(x^3+x^2+x)} \\
 2x^2+2x+2 \\
 \underline{-(2x^2+2x+2)} \\
 0
 \end{array}$$

$$(x+2)(x^2+x+1) = x^3 + 3x^2 + 3x + 2$$

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$$(x^3 + 3x^2 - 13x - 15) \div (x^2 - 2x - 3)$$

$$\begin{array}{r} \underline{x^2 - 2x - 3} \overline{) x^3 + 3x^2 - 13x - 15} \\ \underline{-(x^3 - 2x^2 - 3x)} \\ 5x^2 - 10x - 15 \\ \underline{-(5x^2 - 10x - 15)} \\ 0 \end{array}$$

$$(x+5)(x^2 - 2x - 3) = x^3 + 3x^2 - 13x - 15$$

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Given that 2 is a zero of $P(x) = x^3 + x - 10$,
 use division to factor $P(x)$.

factor $(x-2) = 0$

$$(x-2)(?) = x^3 + x - 10$$

$$\begin{array}{r} \underline{x-2} \overline{) x^3 + 0x^2 + x - 10} \\ \underline{-(x^3 - 2x^2)} \\ 2x^2 + x \\ \underline{-(2x^2 - 4x)} \\ 5x - 10 \\ \underline{-(5x - 10)} \\ 0 \end{array}$$

$$(x-2)(x^2 + 2x + 5)$$

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Synthetic Division

if divisor: $x - k$

$$(x^3 + x - 10) \div (x - 2)$$

$x - k$
 $k = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 1 & -10 \\ & \downarrow & 2 & 4 & 10 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

$x^2 + 2x + 5$

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$$(2x^3 - 3x^2 - 5x - 12) \div (x - 3)$$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & \downarrow & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & 0 \end{array}$$

$2x^2 + 3x + 4$

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$$(2x^4 - 5x^3 + 7x^2 - 3x + 1) \div (x - 3)$$

$$\begin{array}{r} 3 \overline{) 2 \quad -5 \quad 7 \quad -3 \quad 1} \\ \underline{ \downarrow 6 3 30 81} \\ 2 \quad 1 \quad 10 \quad 27 \quad 82 \end{array}$$

$$2x^3 + x^2 + 10x + 27 + \frac{82}{x-3}$$

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Factor Thm

$x - r$ is a factor of a polynomial iff

$$P(r) = 0.$$

Determine if $x + 2$ is a factor of $x^3 - 2x^2 - 5x + 6$.

$$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6 =$$

$$-8 - 8 + 10 + 6 = 0$$

$x + 2$ is a factor.

May 13-11:13 AM

Is $x+3$ a factor of x^3-3x^2-6x+8 ?

$$P(-3) = (-3)^3 - 3(-3)^2 - 6(-3) + 8$$
$$= -28 \neq 0$$

not a factor

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