

8.2 Rational Fncs & Their Graphs

- Obj:
1. Identify & evaluate rational fncs.
 2. Graph, find domain, identify asymptotes, identify holes in the graph.

Rational Fnc: quotient of 2 polynomials

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Find the domain: $g(x) = \frac{x^2 - 7x + 12}{x^2 + 9x + 20}$

$$D: x^2 + 9x + 20 \neq 0$$

$$(x+4)(x+5) \neq 0$$

$$\begin{array}{cc} x+4 \neq 0 & x+5 \neq 0 \\ -4 & -5 \end{array}$$

$$x \neq -4 \quad x \neq -5 \quad \text{domain restriction}$$

$$(-\infty, -5) \cup (-5, -4) \cup (-4, \infty)$$

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Find the domain:

$$j(x) = \frac{3x^2 + x - 2}{x^2 + 2x - 3}$$

$$\begin{aligned} D: x^2 + 2x - 3 &\neq 0 \\ (x-1)(x+3) &\neq 0 \\ x-1 &\neq 0 \quad x+3 \neq 0 \\ x &\neq 1 \quad x \neq -3 \end{aligned}$$

Vertical asymptotes:
 $x=1$ $x=-3$

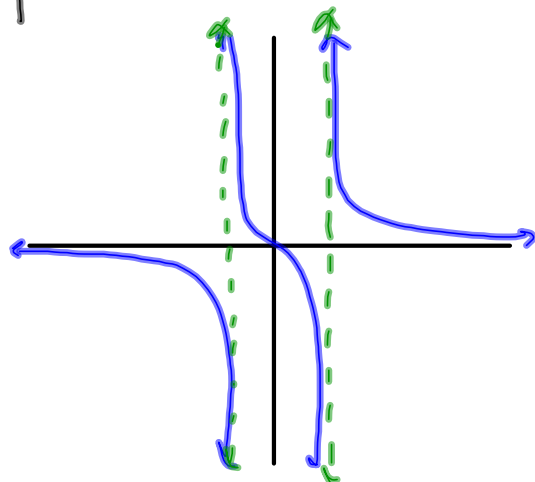
$$(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$$

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Identify vertical asymptotes:

$$r(x) = \frac{2x}{x^2 - 1}$$

$$\begin{aligned} VA: x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$



$$\begin{aligned} D: x &\neq \pm 1 \\ &(-\infty, -1) \cup (-1, 1) \cup (1, \infty) \end{aligned}$$

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Identify VA : $r(x) = \frac{x}{x^2+5x+6}$

VA : $x^2+5x+6=0$
 $(x+3)(x+2)=0$
 $x=-3 \quad x=-2$

D: $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

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Horizontal Asymptotes

$$R(x) = \frac{P}{Q}$$

If: $\deg P < \deg Q$, HA @ $y = 0$

$\deg P = \deg Q$, HA @ $y = \frac{a}{b}$
 (a, b are leading coefficients)

$\deg P > \deg Q$, no HA

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$$R(x) = \frac{x^1}{x^2 - 2x - 3} \begin{matrix} \text{deg 1} \\ \text{deg 2} \end{matrix} \text{ Find VA \& HA.}$$

$$\begin{aligned} \text{VA: } x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x-3=0 \quad x+1 &= 0 \\ x=3 \quad x &= -1 \end{aligned}$$

$$\text{HA: } y = 0$$

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$$\text{Find VA \& HA } R(x) = \frac{2x^2 - 2x + 1}{x^2 - x - 12} \begin{matrix} \text{deg 2} \\ \text{deg 2} \end{matrix}$$

$$\begin{aligned} \text{VA: } x^2 - x - 12 &= 0 \\ (x-4)(x+3) &= 0 \\ x-4=0 \quad x+3 &= 0 \\ x=4 \quad x &= -3 \end{aligned}$$

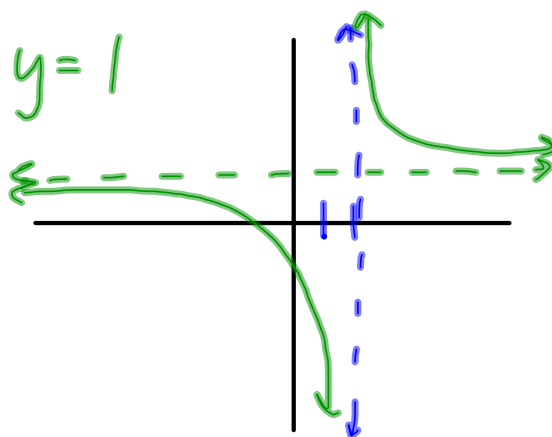
$$\begin{aligned} \text{HA: } y &= \frac{a}{b} \\ y &= 2 \end{aligned}$$

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Graph $y = \frac{|x+2|}{|x-2|}$ showing asymptotes.

VA: $x-2=0$
 $x=2$

HA: $y=1$

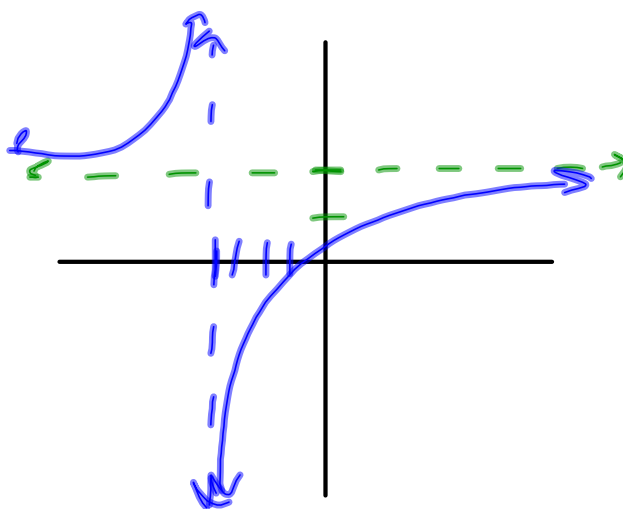


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Graph: $y = \frac{2x-1}{x+4}$

VA: $x+4=0$
 $x=-4$

HA: $y=2$



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Holes

$$y = \frac{x^2 - 9}{x - 3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$$

If $x-b$ is a factor of the denominator & the numerator, then there is a hole in the graph @ $x=b$.

VA: ~~$x-3=0$~~
 ~~$x=3$~~

Hole: $x-3=0$
 $x=3$

HA: none

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Identify all asymptotes & holes in the graph:

$$y = \frac{x^3 + 3x^2}{x^2 + 2x - 3} = \frac{x^2(x+3)}{(x+3)(x-1)}$$

VA: $x^2 + 2x - 3 = 0$
 $(x+3)(x-1) = 0$
 ~~$x = -3$~~ $x = 1$

Holes: $x+3 = 0$
 $x = -3$

HA: none

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