9.2 The Binomial Thm.

Obj: 1. Expand a binomial (2 terms) using Pascal's triangle
2. Find a coefficient of a given term using the Binomial Thm.

Expand

$(a+b)^0$ 1

$(a+b)^1$ $1a^1b^0 + 1b^1a^0$

$(a+b)^2$ $1a^2b^0 + 2a^1b^1 + 1b^2a^0$

$(a+b)^3$ $1a^3b^0 + 3a^2b^1 + 3a^1b^2 + 1b^3a^0$

$(a+b)^n$

patterns: 1. each term has exponents that add up to n.
2. powers of a start at n and decrease by 1 until exp = 0
3. powers of b start at 0 & increase by 1 until exp = n

Coefficients

row 0 $(a+b)^0$ 1

row 1 1 1

row 2 1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

patterns: 1. 1st & last coeffs. are 1

2. Each new row is the sum of the coef. above.

expand $(a+b)^6$ using Pascal's Δ.

row 6: 1 6 15 20 15 6 1
(coefficients)

$1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5$
$+ b^6$

Expand $(p+q)^8$ using Pascal's Δ.

row 8 : 1 8 28 56 70 56 28 8 1

$$p^8 + 8p^7q + 28p^6q^2 + 56p^5q^3 + 70p^4q^4 +$$
$$56p^3q^5 + 28p^2q^6 + 8pq^7 + q^8$$

The coefficients in the expansion $(a+b)^n$ are the values of $_nC_r$ for $r = 0, 1, 2, 3, \ldots n$

Expand $(a+b)^{5}$ using combinations.

n = 5

$_5C_0 = 1$ $\quad$ $_5C_3 = 10$

$_5C_1 = 5$ $\quad$ $_5C_4 = 5$

$_5C_2 = 10$ $\quad$ $_5C_5 = 1$

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

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Expand $(x+y)^7$ using combinations.

$_7C_0 = 1$
$_7C_1 = 7$
$_7C_2 = 21$
$_7C_3 = 35$
$_7C_4 = 35$
$_7C_5 = 21$
$_7C_6 = 7$
$_7C_7 = 1$

$x^7 + 7x^6y + 21x^5y^2 +$
$35x^4y^3 + 35x^3y^4$
$+ 21x^2y^5 + 7xy^6 + y^7$

Expand $(\underset{a}{x}+\underset{b}{2})^8$.

row 8: 1 8 28 56 70 56 28 8 1

$1x^8 + 8x^7(2) + 28x^6(2^2) + 56x^5(2^3) + \ldots\ldots$

$x^8 + 16x^7 + 112x^6 + 448x^5 + \ldots$

Find the coefficient of the x^{10} term in the expansion of $(x+2)^{15}$

$$\underline{{}_{15}C_{10}}\; x^{10} \cdot 2^{5}$$

$$\binom{15}{10} x^{10} \cdot 2^{5}$$

$$\underline{3003} x^{10} \cdot \underline{2^{5}} \quad = \underline{96096} x^{10}$$

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Find the coef. of x^7 in the expansion of $(x-3)^{11}$

$$\binom{11}{7} x^7 (-3)^4$$

$$330x^7(81) = \underline{26730}x^7$$

The Binomial Thm

For any positive integer n:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots\dots + \binom{n}{n}b^n$$

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Use the Binomial Thm to expand $(\underset{a}{2x}-\underset{b}{y^2})^4$

$$=\binom{4}{0}(2x)^4+\binom{4}{1}(2x)^3(-y^2)$$
$$+\binom{4}{2}(2x)^2(-y^2)^2+\ \cdots\cdots$$