

$$-6, -2, 2, 6, 10$$

$\overset{+4}{\curvearrowright}$ $\overset{+4}{\curvearrowright}$
(Arrows indicate the difference between consecutive terms)

a. common diff: 4

b. 10th term: $a_{10} = -6 + 4(9) = 30$

c. recursive: $a_1 = -6$
 $a_n = a_{n-1} + 4$

d. explicit rule: $a_n = a_1 + d(n-1)$
 $a_n = -6 + 4(n-1) = 4n - 10$

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$$6, 9, 12, 15, \dots$$

$\overset{\curvearrowright}$ $\overset{\curvearrowright}$ $\overset{\curvearrowright}$
(Arrows indicate the difference between consecutive terms)

a. $d = a_2 - a_1 = 9 - 6 = 3$

b. $a_{10} = 6 + 3(9) = 33$

c. $a_1 = 6$
 $a_n = a_{n-1} + 3$

d. $a_n = 6 + 3(n-1)$
 $= 6 + 3n - 3 = 3n + 3$

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Geometric Sequence

$$\{a_1, a_1 \cdot r, a_1 \cdot r^2, a_1 \cdot r^3, \dots\}$$

r : common ratio

Recursive

$$a_1 =$$

$$a_n = a_{n-1} \cdot r$$

Explicit

$$a_n = a_1 \cdot r^{n-1}$$

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$$3, 6, 12, 24, 48, \dots$$

a. common ratio: $\frac{a_2}{a_1} = \frac{6}{3} = 2$

b. $a_{10} = 3 \cdot 2^9 = 1536$

c. recursive: $a_1 = 3$
 $a_n = a_{n-1} \cdot 2$

d. explicit: $a_n = 3 \cdot 2^{n-1}$

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9.5 Series

Obj: 1. Use Sigma notation & find sums of sequences.

2. Find sums of convergent geometric series.

$$\text{Seq: } \left\{ \frac{1}{k} : k=1, 2, 3, 4, \dots \right\}$$

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

$$\text{Series: } 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

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Summation/Sigma Notation

The sum of the terms of the sequence

$$\{a_1, a_2, a_3, \dots, a_n\}$$

is:

$$\sum_{k=1}^n a_k$$

"the sum of a_k from $k=1$ to n "

k : index of summation.

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Find the sum:

$$\sum_{k=1}^5 3k = 3 + 6 + 9 + 12 + 15 = 45$$

$$\sum_{k=5}^8 k^2 = 25 + 36 + 49 + 64 = 174$$

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Let $\{a_1, a_2, a_3, \dots, a_n\}$
 be a finite arithmetic sequence with
 common diff. d

$$\begin{aligned} \text{Sum} &= \sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n \\ &= n \left(\frac{a_1 + a_n}{2} \right) \\ &= \frac{n}{2} (2a_1 + (n-1)d) \end{aligned}$$

n : # of terms

a_1 : 1st term

a_n : last term

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Find the sum: $-8, \dots, 27$
 (arithmetic) $d=7$

$$\text{Sum: } n \left(\frac{a_1 + a_n}{2} \right) = 6 \left(\frac{-8 + 27}{2} \right) = 57$$

$a_1 = -8$
 $a_n = 27$

~~$a_n = a_{n-1} + d$~~
 explicit
 $a_n = a_1 + d(n-1)$
 $27 = -8 + d(n-1)$
 $27 = -8 + 7(n-1)$
 $n = 6$

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Let $\{a_1, a_2, a_3, \dots, a_n\}$ be a finite
geometric sequence. common ratio: r

$$\text{Sum} = \sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$$

$$= \frac{a_1 (1 - r^n)}{1 - r}$$

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Find the sum: 5, 15, 45, ..., 98415

$$\text{Sum: } \frac{a_1(1-r^n)}{1-r}$$

$a_1 = 5$
 $r = 3$
 $n =$

explicit: $a_n = a_1 \cdot r^{n-1}$

$$98415 = 5 \cdot 3^{n-1}$$

$$\frac{98415}{5} = 3^{n-1}$$

$$\ln 19683 = (n-1) \ln 3$$

$$9 = n-1$$

$$n = 10$$

$$\text{Sum} = \frac{5(1-3^{10})}{1-3}$$

$$= 147620$$

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Infinite Series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

Sometimes the sum can approach a finite limit.

The geometric series $\sum_{k=1}^{\infty} a \cdot r^{k-1}$

converges iff $|r| < 1$

* If it converges:

$$\text{Sum} = \frac{a_1}{1-r}$$

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Determine if it conv. If so, find sum

$$\sum_{k=1}^{\infty} 3 \left(\frac{.75}{r} \right)^{k-1}$$

$r < 1 \rightarrow \text{conv.}$

$$\text{Sum: } \frac{a_1}{1-r} = \frac{3}{1-.75} = 12$$

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