## Section 1.2 Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, **(a)** write a formula for the function and **(b)** use the formula to find the indicated value of the function.

**1.** the area $A$ of a circle as a function of its diameter $d$; the area of a circle of diameter 4 in. (a) $A(d) = \pi\left(\frac{d}{2}\right)^2$ (b) $A(4) = 4\pi$ in$^2$

**2.** the height $h$ of an equilateral triangle as a function of its side length $s$; the height of an equilateral triangle of side length 3 m

**3.** the surface area $S$ of a cube as a function of the length of the cube's edge $e$; the surface area of a cube of edge length 5 ft

**4.** the volume $V$ of a sphere as a function of the sphere's radius $r$; the volume of a sphere of radius 3 cm

In Exercises 5–12, **(a)** identify the domain and range and **(b)** sketch the graph of the function.

**5.** $y = 4 - x^2$ $(-\infty, \infty)$; $(-\infty, 4]$

**6.** $y = x^2 - 9$ $(-\infty, \infty)$; $[-9, \infty)$

**7.** $y = 2 + \sqrt{x - 1}$ $[1, \infty)$; $[2, \infty)$

**8.** $y = -\sqrt{-x}$ $(-\infty, 0]$; $(-\infty, 0]$

**9.** $y = \dfrac{1}{x - 2}$ $(-\infty, 2) \cup (2, \infty)$; $(-\infty, 0) \cup (0, \infty)$

**10.** $y = \sqrt[4]{-x}$ $(-\infty, 0]$; $[0, \infty)$

**11.** $y = 1 + \dfrac{1}{x}$ $(-\infty, 0) \cup (0, \infty)$; $(-\infty, 1) \cup (1, \infty)$

**12.** $y = 1 + \dfrac{1}{x^2}$ $(-\infty, 0) \cup (0, \infty)$; $(1, \infty)$

In Exercises 13–20, use a grapher to **(a)** identify the domain and range and **(b)** draw the graph of the function.

**13.** $y = \sqrt[3]{x}$ $(-\infty, \infty)$; $(-\infty, \infty)$

**14.** $y = 2\sqrt{3 - x}$ $(-\infty, 3]$; $[0, \infty)$

**15.** $y = \sqrt[3]{1 - x^2}$ $(-\infty, \infty)$; $(-\infty, 1]$

**16.** $y = \sqrt{9 - x^2}$ $[-3, 3]$; $[0, 3]$

**17.** $y = x^{2/5}$ $(-\infty, \infty)$; $[0, \infty)$

**18.** $y = x^{3/2}$ $[0, \infty)$; $[0, \infty)$

**19.** $y = \sqrt[3]{x - 3}$ $(-\infty, \infty)$; $(-\infty, \infty)$

**20.** $y = \dfrac{1}{\sqrt{4 - x^2}}$ $(-2, 2)$; $[0.5, \infty)$

In Exercises 21–30, determine whether the function is even, odd, or neither.

**21.** $y = x^4$ Even

**22.** $y = x + x^2$ Neither

**23.** $y = x + 2$ Neither

**24.** $y = x^2 - 3$ Even

**25.** $y = \sqrt{x^2 + 2}$ Even

**26.** $y = x + x^3$ Odd

**27.** $y = \dfrac{x^3}{x^2 - 1}$ Odd

**28.** $y = \sqrt[3]{2 - x}$ Neither

**29.** $y = \dfrac{1}{x - 1}$ Neither

**30.** $y = \dfrac{1}{x^2 - 1}$ Even

In Exercises 31–34, graph the piecewise-defined functions.

**31.** $f(x) = \begin{cases} 3 - x, & x \le 1 \\ 2x, & 1 < x \end{cases}$

**32.** $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$

**33.** $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ (3/2)x + 3/2, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$

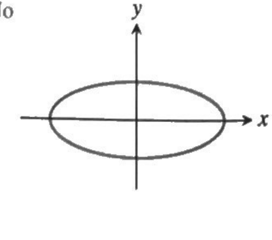
**34.** $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x - 1, & x > 1 \end{cases}$

**35. Writing to Learn** The *vertical line test* to determine whether a curve is the graph of a function states: If every vertical line in the $xy$-plane intersects a given curve in at most one point, then the curve is the graph of a function. Explain why this is true.

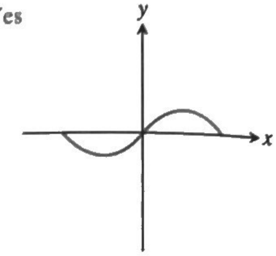
**36. Writing to Learn** For a curve to be *symmetric about the x-axis*, the point $(x, y)$ must lie on the curve if and only if the point $(x, -y)$ lies on the curve. Explain why a curve that is symmetric about the $x$-axis is not the graph of a function, unless the function is $y = 0$.

In Exercises 37–40, use the vertical line test (see Exercise 35) to determine whether the curve is the graph of a function.

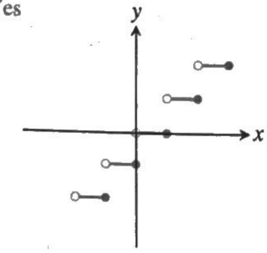
**37.** No



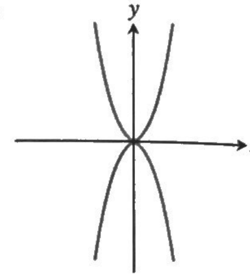

**38.** Yes




**39.** Yes

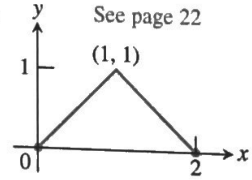



**40.** No

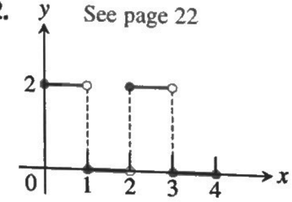



In Exercises 41–48, write a piecewise formula for the function.

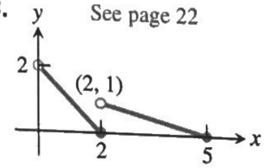
**41.** See page 22



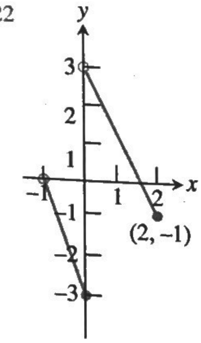

**42.** See page 22




**43.** See page 22

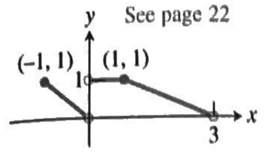



**44.** See page 22




**2.** (a) $h(s) = \dfrac{\sqrt{3}}{2}s$ (b) $h(3) = 3\dfrac{\sqrt{3}}{2}$ m **3.** (a) $S(e) = 6e^2$ (b) $S(5) = 150$ ft$^2$ **4.** (a) $V(r) = \dfrac{4}{3}\pi r^3$ (b) $V(3) = 36\pi$ cm$^3$

45. See page 22

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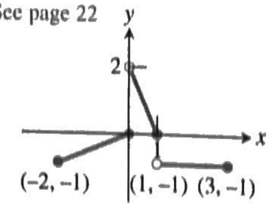


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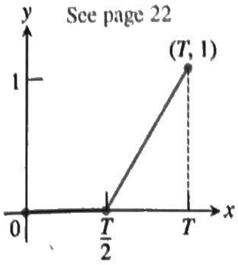
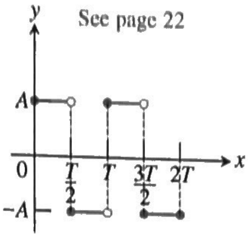



48. See page 22




In Exercises 49 and 50, **(a)** draw the graph of the function. Then find its **(b)** domain and **(c)** range. 49. (b) All reals (c) $(-\infty, 2]$

**49.** $f(x) = -|3 - x| + 2$

**50.** $f(x) = 2|x + 4| - 3$
(b) All reals (c) $[-3, \infty)$

In Exercises 51 and 52, find

**(a)** $f(g(x))$ **(b)** $g(f(x))$ **(c)** $f(g(0))$
**(d)** $g(f(0))$ **(e)** $g(g(-2))$ **(f)** $f(f(x))$

**51.** $f(x) = x + 5, \quad g(x) = x^2 - 3$ (a) $x^2 + 2$ (b) $x^2 + 10x + 22$ (c) 2 (d) 22 (e) $-2$ (f) $x + 10$

**52.** $f(x) = x + 1, \quad g(x) = x - 1$ (a) $x$ (b) $x$ (c) 0 (d) 0 (e) $-4$ (f) $x + 2$

**53.** Copy and complete the following table.

| | $g(x)$ | $f(x)$ | $(f \circ g)(x)$ |
|---|---|---|---|
| (a) | ? $g(x) = x^2$ | $\sqrt{x - 5}$ | $\sqrt{x^2 - 5}$ |
| (b) | ? | $1 + 1/x$ | $x$ $g(x) = \frac{1}{x-1}$ |
| (c) | $1/x$ | ? $f(x) = \frac{1}{x}$ | $x$ |
| (d) | $\sqrt{x}$ | ? $f(x) = x^2$ | $\lvert x \rvert, x \geq 0$ |

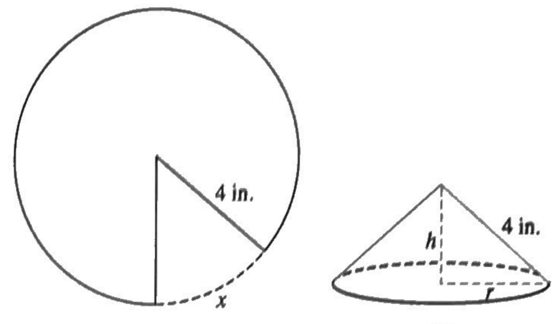
**54. *The Cylindrical Can Problem*** A cylindrical can is to be constructed so that its height $h$ is equal to its diameter.

(a) Write the volume of the cylinder as a function of its height $h$.

(b) Write the total surface area of the cylindrical can (including the top and bottom) as a function of its height $h$.

(c) Find the volume of the can if its total surface area is $54\pi$ square inches.

**55. *The Cone Problem*** Begin with a circular piece of paper with a 4-in. radius as shown in (a). Cut out a sector with an arc length of $x$. Join the two edges of the remaining portion to form a cone with radius $r$ and height $h$, as shown in (b).




(a) (b)

**(a)** Explain why the circumference of the base of the cone is $8\pi - x$.

**(b)** Express the radius $r$ as a function of $x$.

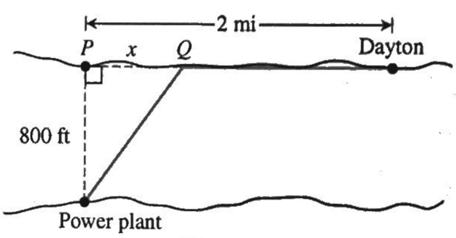
**(c)** Express the height $h$ as a function of $x$.

**(d)** Express the volume $V$ of the cone as a function of $x$.

**56. *Industrial Costs*** Dayton Power and Light, Inc., has a power plant on the Miami River where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs \$180 per foot across the river and \$100 per foot along the land.

**(a)** Suppose that the cable goes from the plant to a point $Q$ on the opposite side that is $x$ ft from the point $P$ directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance $x$.

**(b)** Generate a table of values to determine if the least expensive location for point $Q$ is less than 2000 ft or greater than 2000 ft from point $P$.




(Not to scale)

56. (b)
$C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) \approx \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from point $P$.

## Standardized Test Questions

**57. True or False** The function $f(x) = x^4 + x^2 + x$ is an even function. Justify your answer. False. $f(-x) \neq f(x)$

**58. True or False** The function $f(x) = x^{-3}$ is an odd function. Justify your answer. True. $f(-x) = -f(x)$

54. (a) $V = \pi r^2 h = \pi\left(\frac{h}{2}\right)^2 \cdot h = \frac{\pi h^3}{4}$

(b) $A = 2\pi r h + 2 \cdot \pi r^2 = 2\pi\left(\frac{h}{2}\right)h + 2\pi\left(\frac{h}{2}\right)^2 = \frac{3\pi h^2}{2}$

(c) $54\pi = \frac{3\pi h^2}{2} \Rightarrow h = 6 \Rightarrow V = \frac{\pi 6^3}{4} = 54\pi$

55. (a) Because the circumference of the original circle was $8\pi$ and a piece of length $x$ was removed. (b) $r = \frac{8\pi - x}{2\pi} = 4 - \frac{x}{2\pi}$

(c) $h = \sqrt{16 - r^2} = \frac{\sqrt{16\pi x - x^2}}{2\pi}$

(d) $V = \frac{1}{3}\pi r^2 h = \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$

56. (a) $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$

**59. Multiple Choice** Which of the following gives the domain of

$f(x) = \dfrac{x}{\sqrt{9 - x^2}}$? B

(A) $x \neq \pm 3$ (B) $(-3, 3)$ (C) $[-3, 3]$

(D) $(-\infty, -3) \cup (3, \infty)$ (E) $(3, \infty)$

**60. Multiple Choice** Which of the following gives the range of

$f(x) = 1 + \dfrac{1}{x - 1}$? A

(A) $(-\infty, 1) \cup (1, \infty)$ (B) $x \neq 1$ (C) all real numbers

(D) $(-\infty, 0) \cup (0, \infty)$ (E) $x \neq 0$

**61. Multiple Choice** If $f(x) = 2x - 1$ and $g(x) = x + 3$, which of the following gives $(f \circ g)(2)$? D

(A) 2 (B) 6 (C) 7 (D) 9 (E) 10

**62. Multiple Choice** The length $L$ of a rectangle is twice as long as its width $W$. Which of the following gives the area $A$ of the rectangle as a function of its width? C

(A) $A(W) = 3W$ (B) $A(W) = \dfrac{1}{2}W^2$

(C) $A(W) = 2W^2$ (D) $A(W) = W^2 + 2W$

(E) $A(W) = W^2 - 2W$

## Explorations

In Exercises 63–66, **(a)** graph $f \circ g$ and $g \circ f$ and make a conjecture about the domain and range of each function. **(b)** Then confirm your conjectures by finding formulas for $f \circ g$ and $g \circ f$.

**63.** $f(x) = x - 7,\quad g(x) = \sqrt{x}$ $(f \circ g)(x) = \sqrt{x} - 7$; $(g \circ f)(x) = \sqrt{x - 7}$

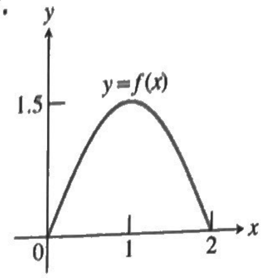
**64.** $f(x) = 1 - x^2,\quad g(x) = \sqrt{x}$ $(f \circ g)(x) = 1 - (\sqrt{x})^2 = 1 - x, x \geq 0$

**65.** $f(x) = x^2 - 3,\quad g(x) = \sqrt{x + 2}$ $(g \circ f)(x) = \sqrt{1 - x^2}$

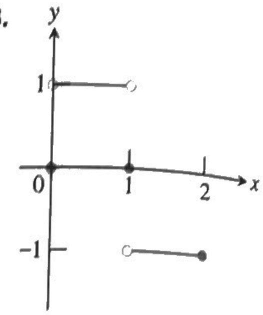
**66.** $f(x) = \dfrac{2x - 1}{x + 3},\quad g(x) = \dfrac{3x + 1}{2 - x}$

**Group Activity** In Exercises 67–70, a portion of the graph of a function defined on $[-2, 2]$ is shown. Complete each graph assuming that the graph is **(a)** even, **(b)** odd.

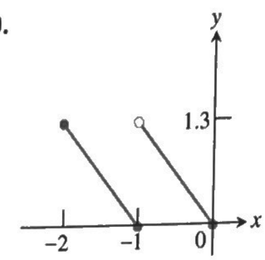
**67.**



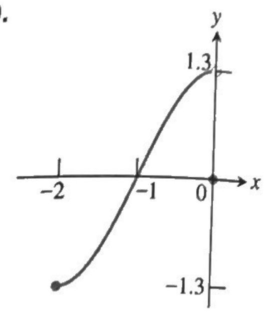

**68.**




**69.**




**70.**




## Extending the Ideas

**71.** Enter $y_1 = \sqrt{x}$, $y_2 = \sqrt{1 - x}$ and $y_3 = y_1 + y_2$ on your grapher.

**(a)** Graph $y_3$ in $[-3, 3]$ by $[-1, 3]$.

**(b)** Compare the domain of the graph of $y_3$ with the domains of the graphs of $y_1$ and $y_2$.

**(c)** Replace $y_3$ by

$y_1 - y_2,\quad y_2 - y_1,\quad y_1 \cdot y_2,\quad y_1/y_2,\quad$ and $\quad y_2/y_1,$

in turn, and repeat the comparison of part (b).

**(d)** Based on your observations in (b) and (c), what would you conjecture about the domains of sums, differences, products, and quotients of functions?

**72. *Even and Odd Functions***

**(a)** Must the product of two even functions always be even? Give reasons for your answer.

**(b)** Can anything be said about the product of two odd functions? Give reasons for your answer.

41. $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2 \end{cases}$

42. $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$

43. $f(x) = \begin{cases} 2 - x, & 0 < x \leq 2 \\ \dfrac{5}{3} - \dfrac{x}{3}, & 2 < x \leq 5 \end{cases}$

44. $f(x) = \begin{cases} -3x - 3, & -1 < x \leq 0 \\ -2x + 3, & 0 < x \leq 2 \end{cases}$

45. $f(x) = \begin{cases} -x, & -1 \leq x < 0 \\ 1, & 0 < x \leq 1 \\ \dfrac{3}{2} - \dfrac{x}{2}, & 1 < x < 3 \end{cases}$

46. $f(x) = \begin{cases} \dfrac{x}{2}, & -2 \leq x \leq 0 \\ -2x + 2 & 0 < x \leq 1 \\ -1, & 1 < x \leq 3 \end{cases}$

47. $f(x) = \begin{cases} 0, & 0 \leq x \leq \dfrac{T}{2} \\ \dfrac{2}{T}x - 1, & \dfrac{T}{2} < x \leq T \end{cases}$

48. $f(x) = \begin{cases} A, & 0 \leq x < \dfrac{T}{2} \\ -A, & \dfrac{T}{2} \leq x < T \\ A, & T \leq x < \dfrac{3T}{2} \\ -A, & \dfrac{3T}{2} \leq x \leq 2T \end{cases}$

65. (b) $(f \circ g)(x) = (\sqrt{x + 2})^2 - 3 = x - 1, x \geq -2$

$(g \circ f)(x) = \sqrt{x^2 - 1}$

66. (b) $(f \circ g)(x) = x, x \neq 2$

$(g \circ f)(x) = x, x \neq -3$