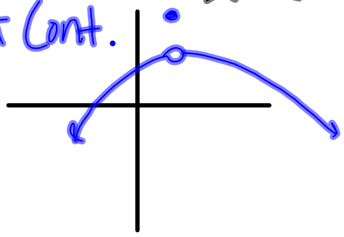
# Continuity

$f(x)$ is cont. @ $x=c$ if:

1. $f(c)$ exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Not Cont.



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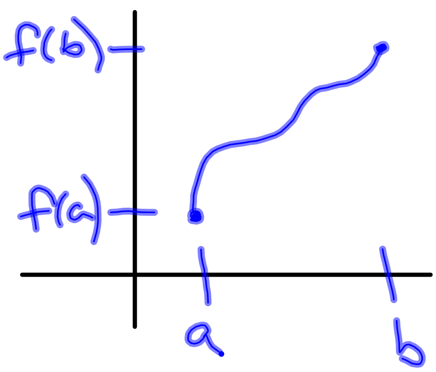
# IVT

If $f(x)$ is continuous on $[a,b]$ then the fnc. takes on every y value from $f(a)$ to $f(b)$.

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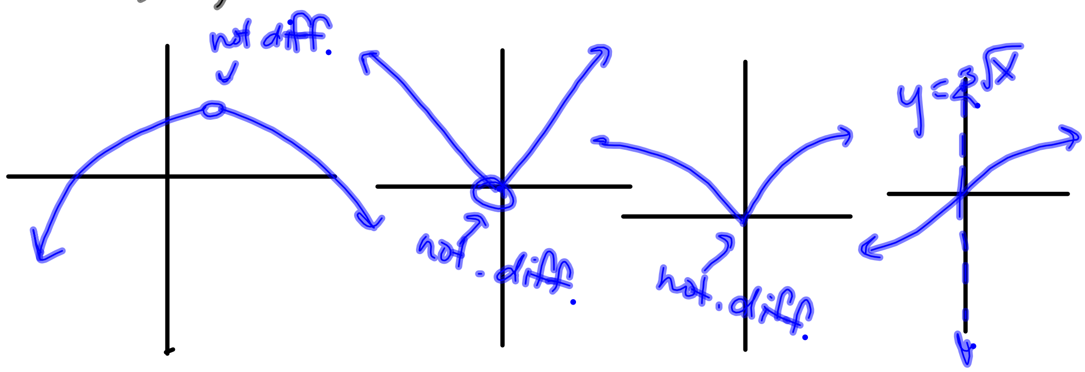
## Differentiability

If a fnc. is differentiable, then it is continuous.

$f(x)$ is diff. if LHD = RHD.

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CP: $f'(x)=0$

local max/min of $f(x)$

→ must have a sign change!

Inflection Points:

$f''(x)=0$

$f'$ has max/min

$f(x)$ changes concavity

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Let f be the function given by $f(x)=2xe^{2x}$.

a) Find the absolute minimum of f. Justify that your answer is an absolute minimum.

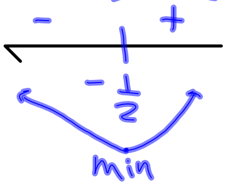
$f'(x)=0$

$f'(x)=2x\cdot e^{2x}\cdot 2+2\cdot e^{2x}=0$

$2e^{2x}(2x+1)=0$

$\cancel{2e^{2x}=0}$ $\quad 2x+1=0$

$x=-\frac{1}{2}$




$f(-\frac{1}{2})=-e^{-1}=-\frac{1}{e}$

f has an abs. min of $-\frac{1}{e}$ which occurs @ $x=-\frac{1}{2}$. It is an abs. min b/c there is no other CP. and b/c it is where f' changes from neg to pos & where f changes from dec. to inc.

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MVT (for derivatives)

If $f(x)$ is differentiable over $[a,b]$ then there exists a point $x=c$ on $[a,b]$ when $f'(c)=\frac{f(b)-f(a)}{b-a}$.

instantaneous rate of change, slope of tangent line. — slope of secant line, average rate of change

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## Fundamental Thm of Calculus:

$$\frac{d}{dx}\int_c^{x} f(t)\,dt = f(x)$$

Find $\frac{dy}{dx}$:

1. $y=\int_\pi^{x} \cos t\,dt$

$\frac{dy}{dx} = \cos x \cdot 1$

Long way:

$y = \sin t \Big|_\pi^x$

$= \sin x - \cancel{\sin \pi}$

$y = \sin x$

$\frac{dy}{dx} = \cos x$

2. $y=\int_4^{x^2} e^t\,dt$

$\frac{dy}{dx} = e^{x^2}\cdot 2x = 2xe^{x^2} - \cancel{e^4}\,0$

3. $y=\int_{2x}^{x^2} \sin t\,dt$

$\frac{dy}{dx} = 2x\sin x^2 - 2\sin 2x$

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## Average Value

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

## MVT (for integrals)

If $f$ is integrable on $[a,b]$, the average value will equal the fnc at some $x=c$.

$av(f(x)) = f(x)$ for some $x=c$

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If $f(x) = -2x^3$ $[1,3]$, at what $x$ value does the fnc. take on its av. value?

$$av(f) = \frac{1}{3-1}\int_1^3 -2x^3\,dx$$

$$\frac{1}{2}\left(-\frac{1}{2}x^4\right)\Big|_1^3$$

$$-\frac{1}{4}(3^4 - 1^4) = -20$$

$$\frac{-20}{-2} = \frac{-2x^3}{-2}$$

$$x^3 = 10$$

$$x = \sqrt[3]{10}$$

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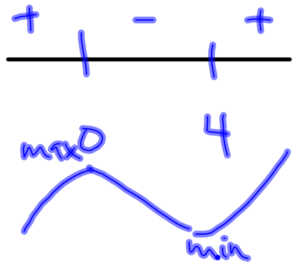
Let f be the function given by $f(x)=x^3-6x^2+p$, where p is an arbitrary constant.

a) Write an expression for f'(x) and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

$$f'(x) = 3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$x = 0, 4$$




$$f(0) = p$$

$$f(4) = 64 - 6(16) + p$$
$$= -32 + p$$

max : $(0, p)$

min : $(4, -32+p)$

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b) Find the value of p such that the average value of f over the closed interval [-1,2] is 1.

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{2+1}\int_{-1}^{2}(x^3-6x^2+p)\,dx$$

$$\frac{1}{3}\left(\frac{1}{4}x^4-2x^3+px\right)\Big|_{-1}^{2}=1$$

$$\left(\frac{1}{4}x^4-2x^3+px\right)\Big|_{-1}^{2}=3$$

$$\frac{1}{4}(16)-2(8)+2p-\left(\frac{1}{4}+2-p\right)=3$$

$$4-16+2p-\frac{1}{4}-2+p=3$$

$$3p-14.25=3$$

$$3p=17.25$$

$$p=5.75$$

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