# U-Substitution

1. Identify the "inside" fnc.
2. Let u = the inside.
3. Find du
4. Get a "match", then substitute.
5. Integrate (w/ respect to u)
6. Substitute back to x.

Apr 28-1:28 PM

1. $\int e^{\cos x} \sin x \, dx$

$u = \cos x$

$-1 \; du = -\sin x \, dx$

$-du = \sin x \, dx$

$\int e^{u}(-du)$

$-1\int e^{u} du$

$= -e^{u} + C$

$= -e^{\cos x} + C$

Apr 28-2:38 PM

2. $\int \frac{\ln^4 x}{2x}dx = \frac{1}{2}\int \frac{\ln^4 x}{x}dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\frac{1}{2}\int u^4 du = \frac{1}{2}\left(\frac{1}{5}u^5\right) + C$

$= \frac{1}{10}(\ln x)^5 + C$

Apr 28-2:40 PM

3. $\int_1^2 \frac{2x}{(x^2+3)^2}dx$

$u = x^2+3$

$du = 2x\,dx$

$\int \frac{du}{u^2}$ or $\int_4^7 u^{-2}du$

$= -u^{-1}\Big|_4^7 = -\left(7^{-1} - 4^{-1}\right)$

$-(x^2+3)^{-1}\Big|_1^2$

$= -\left(\frac{1}{7} - \frac{1}{4}\right) = \frac{3}{28}$

Apr 28-2:46 PM

# Initial Value Problems

1. Separate Variables
2. Integrate both sides
3. Substitute to find C
4. Plug in C → solve for y.

Apr 28-2:49 PM

Find y if $\frac{dy}{dx} = 4xy$

and $y = e^4$ when $x = 1$.

$$\int \frac{dy}{y} = \int 4x\,dx$$

$$\ln|y| = 2x^2 + C$$

$$\ln e^4 = 2 + C$$

$$4 = 2 + C$$

$$C = 2$$

$$\ln y = 2x^2 + 2$$

$$e^{2x^2+2} = y$$

$$y = e^{2x^2+2}$$

Apr 28-2:51 PM

$$\frac{dy}{dx} = 3y \qquad y=10 \text{ when } x=0$$

$$\int \frac{dy}{y} = \int 3\,dx$$

$$\ln|y| = 3x + C$$

$$\ln 10 = C$$

$$\ln|y| = 3x + \ln 10$$

$$e^{3x+\ln 10} = y$$

$$e^{3x} \cdot e^{\ln 10} = y$$

$$y = 10e^{3x}$$

Apr 28-2:54 PM

Let v(t) be the velocity, in feet per second, of a skydiver at time t seconds. After her parachute opens, her velocity satisfies the differential equation dv/dt= -2v-32, with initial condition v(0)= -50.

a) Use separation of variables to find an expression for v in terms of t, where t is measures in seconds.

$$\frac{dv}{dt} = -2v-32 = -2(v+16)$$

$$\int \frac{dv}{v+16} = \int -2\,dt$$

$$\ln|v+16| = -2t + C$$

$$\ln|-50+16| = -2\cdot 0 + C$$

$$C = \ln 34$$

$$\ln|v+16| = -2t+\ln 34$$

$$e^{-2t+\ln 34} = v+16$$

$$V = 34e^{-2t} - 16$$

Apr 28-1:10 PM

b) Terminal velocity is defined as the limit of v(t) as t approaches infinity. Find the terminal velocity of the skydiver to the nearest foot per second.

$$\lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(34e^{-2t}-16\right)$$

$$= -16$$

Apr 28-1:26 PM

## Movement of a Particle

Position: $s(t)$ or $x(t)$

Instantaneous Velocity: $v(t) = s'(t)$

Acceleration: $a(t) = s''(t) = v'(t)$

Displacement: $\Delta s$ = final pos - starting pos

Total Dist: $\int |v(t)|\,dt$

Speeding up: $v(t)$ & $a(t)$ have same sign

slowing down: $v(t)$ & $a(t)$ have opposite signs.

Average velocity: $\frac{\Delta s}{\Delta t} = \frac{s(t_2)-s(t_1)}{t_2-t_1}$

Apr 28-3:12 PM

$s(t) = t^3 - 2t^2 - 4t + 2$ in feet & seconds

Find $v(3)$ & $a(t)$

$v(t) = 3t^2 - 4t - 4$

$v(3) = 27 - 12 - 4 = 11$ ft/sec.

$a(t) = 6t - 4$

$a(3) = 18 - 4 = 14$ ft/sec$^2$

@ $t = 3$ sec. the particle is speeding up because both $v(t)$ & $a(t)$ are positive.

Apr 28-3:15 PM

A particle moves along the x-axis so that its velocity at any time (t greater than or equal to zero) is given by $v(t)=3t^2-2t-1$. The position x(t) is 5 for t=2.

a) Write a polynomial expression for the position of the particle at any time t.

$x(t) = \int v(t)dt = t^3 - t^2 - t + C$

$5 = 8 - 4 - 2 + C$

$C = 3$

$x(t) = t^3 - t^2 - t + 3$

b) For what values of t on [0,3], is the particle's instantaneous velocity the same as its average velocity on the closed interval [0,3].

avg. vel. $\dfrac{x(3) - x(0)}{3 - 0} = \dfrac{18 - 3}{3} = 5$

$5 = 3t^2 - 2t - 1$

$3t^2 - 2t - 6 = 0$

$t = \dfrac{2 \pm \sqrt{4 - 4(3)(-6)}}{6} = \dfrac{1 \pm \sqrt{19}}{3}$

Apr 28-12:32 PM

c) Find the total distance traveled by the particle from time t=0 until time t=3.

dist: $\int_0^3 |v(t)|\,dt = 17$

Apr 28-12:44 PM