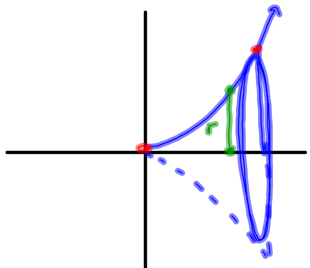
Volumes

$f(x)=x^2 \quad [0,4]$

rotated around the x-axis.



$\to A = \pi r^2 = \pi(x^2)^2$

of crossections

πx^4

$V = \int_0^4 (\pi x^4)\,dx$

$\pi \int_0^4 x^4\,dx = \pi\left(\frac{1}{5}x^5\right)\Big|_0^4$

$= \frac{\pi}{5}(4^5) = \frac{1024\pi}{5}$

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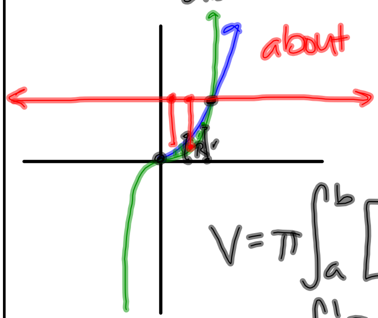
$y=x^2 \quad y=x^3$

about x-axis

about $y=1$



$V=\pi\int_a^b [R^2 - r^2]\,dx$

$V=\pi\int_0^1 [(x^2)^2 - (x^3)^2]\,dx$

$V=\pi\int_0^1 [(1-x^3)^2 - (1-x^2)^2]\,dx$

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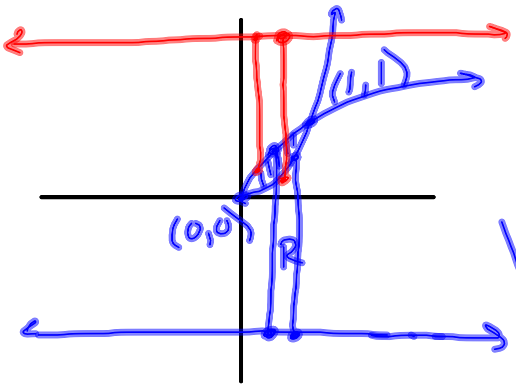
$y=x^3 \quad y=\sqrt{x} \quad$ about $y=-3$

$y=3$



$$V=\pi\int_0^1\left[(\sqrt{x}+3)^2-(x^3+3)^2\right]dx$$

$\vdots$

$$V=\pi\int_0^1\left[(3-x^3)^2-(3-\sqrt{x})^2\right]dx$$

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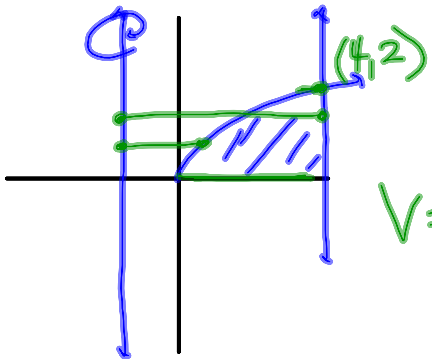
$y=\sqrt{x} \quad y=0 \quad x=4$

$x=y^2$ about $x=-2$



$$V=\pi\int_0^2\left[6^2-(y^2+2)^2\right]dy$$

$\vdots$

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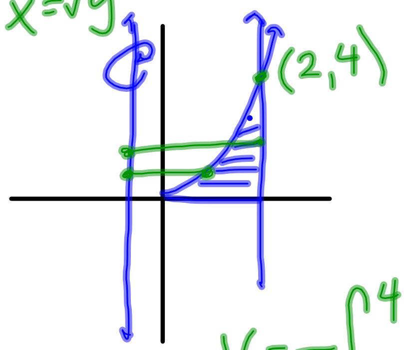
$y=x^2 \quad y=0 \quad x=2 \quad \text{about } x=-1$

$x=\sqrt{y}$



$$V=\pi\int_0^4\left[3^2-(\sqrt{y}+1)^2\right]dy$$

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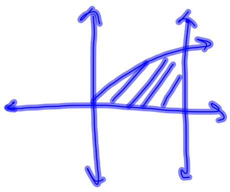
Let R be the region bounded by the x-axis, the graph of $y=\sqrt{x}$, and the line $x=4$.

a) Find the area of the region R.



$$A=\int_0^4\sqrt{x}\,dx=\frac{2}{3}x^{\frac{3}{2}}\Big|_0^4$$

$$\frac{2}{3}\left(4^{\frac{3}{2}}-0^{\frac{3}{2}}\right)=\frac{16}{3}$$

b) Find the value of h such that the vertical line x=h divides the region R into two regions of equal area.

$$\int_0^h\sqrt{x}\,dx=\frac{8}{3}$$

$$\frac{2}{3}x^{\frac{3}{2}}\Big|_0^h=\frac{8}{3}$$

$$\frac{2}{3}h^{\frac{3}{2}}=\frac{8}{3}$$

$$\left(h^{\frac{3}{2}}\right)^{\frac{2}{3}}=(4)^{\frac{2}{3}}$$

$$h=\sqrt[3]{16}$$

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c) Find the volume of the solid generated when R is revolved about the x-axis.

$$V = \pi\int_0^4 \sqrt{x}^2\,dx = \pi\int_0^4 x\,dx$$

$$\pi\left(\tfrac{1}{2}x^2\right)\Big|_0^4 = \frac{\pi}{2}(16-0)$$

$$= 8\pi$$

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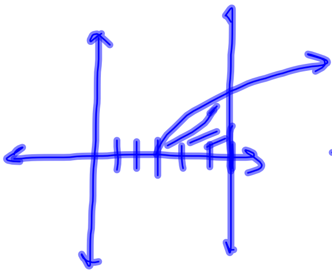
Let R be the region bounded by the x-axis, the function $y=\sqrt{x-3}$, and the vertical line x=6.

a) Find the area of region R.



$$A = \int_3^6 \sqrt{x-3}\,dx$$

$$\frac{2}{3}(x-3)^{\frac{3}{2}}\Big|_3^6 = \frac{2}{3}\left(3^{\frac{3}{2}} - 0^{\frac{3}{2}}\right)$$

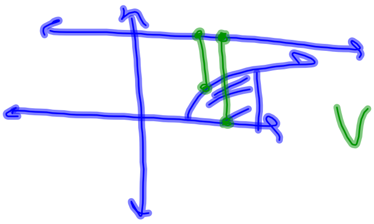
$$\frac{2}{3}\left(3^{\frac{3}{2}}\right)$$

$$\frac{2}{3}\sqrt{27}$$

$$\frac{2}{3}(3\sqrt{3})$$

$$2\sqrt{3}$$

b) Write, but do not evaluate, the integral that represents the volume generated by revolving region R about the horizontal line y=4.



$$V = \pi\int_3^6 \left[4^2 - \left(4-\sqrt{x-3}\right)^2\right]dx$$

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